Dissipationless Transport in Low-Density Bilayer Systems

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In a bilayer electronic system the layer index may be viewed as the z component of an isospin-$\frac{1}{2}$. An XY isospin-ordered ferromagnetic phase was observed in quantum Hall systems and is predicted to exist at zero magnetic field at low density. This phase is a superfluid for opposite currents in the two layers. At $B = 0$ the system is gapless but superfluidity is not destroyed by weak disorder. In the quantum Hall case, weak disorder generates a random gauge field which probably does not destroy superfluidity. Experimental signatures include Coulomb drag and collective mode measurements.

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In quantum well structures containing two separate two-dimensional electron gases in close proximity, an electron is described in terms of its position in the plane, its spin, and its layer index. The latter can be regarded as an isospin-$\frac{1}{2}$, denoted by $\mathbf{m}$, with the two layers being the two eigenstates of m_z. States with spontaneous XY isospin-ferromagnetic order have been observed in quantum Hall systems [1] at total Landau level filling factor $\nu = 1$ and were predicted to exist at $B = 0$ for sufficiently low electron density [2]. The origin of isospin ferromagnetism is a favorable Coulomb exchange energy just as in the ordinary Stoner instability.

For layer separation $d = 0$ the isospin polarized phase breaks an SU(2) symmetry, and the problem maps onto the Stoner instability [1]. For $d > 0$, and in the absence of tunneling between the layers, there is an easy-plane anisotropy since the direct Coulomb energy favors polarization in the XY plane ($\langle m_z \rangle = 0$) in order to avoid the cost of charge imbalance between the layers that occurs for $\langle m_z \rangle \neq 0$. The angle of the magnetization $\mathbf{m}(\mathbf{r})$ relative to the x axis is then described by a field $\varphi(\mathbf{r})$. Because the "charge" conjugate to the phase φ is m_z, the Goldstone mode [3] associated with the broken U(1) symmetry at finite d corresponds to superfluid currents which are *opposite* to each layer [1,4,5].

In this paper we study transport properties of the easy-plane isospin ferromagnet, focusing on the effects of disorder. At $B = 0$ we find that disorder weakens but does not destroy the "gapless isospin" superfluidity. The lack of time-reversal symmetry in the quantum Hall effect (QHE) case causes disorder to induce a random gauge field which frustrates the system, but the Kosterlitz-Thouless transition probably survives weak disorder. The effect of random interlayer tunneling is a separate and different question [6].

For $B = 0$ it is difficult to quantify the range of parameters, particularly r_s, in which the isospin ferromagnetic state is the prevailing phase. It should lie between the low r_s paramagnetic range and the very high r_s range, where the system forms a bilayer Wigner crystal. For a single layer, Monte Carlo calculations [7] find a ferromagnetic transition in the $r_s \sim 20$–30 range, and Wigner crystallization at $r_s \approx 37 \pm 5$ [8]. The energy differences among the various possible phases are, however, very small [7] so that a definitive statement is not possible—in fact, earlier calculations [8] did not find a ferromagnetic transition. For a double layer system, Hartree-Fock (HF) theory predicts the existence of a broken symmetry isospin ferromagnetic phase [2]. HF tends to overestimate the stability of broken symmetry states, but its predictions are often qualitatively correct and such states frequently do occur at values of r_s larger than predicted. While quantum Monte Carlo calculations are needed to obtain the precise density at which a $B = 0$ bilayer system will undergo the spontaneous isospin ferromagnetic transition, it is reasonable to assume, based on existing HF analysis [2], that such a transition should occur at $r_s \approx 20$–30, a regime now realizable in hole systems [9].

An HF analysis of the isospin polarized phase starts with an Hubbard-Stratanovich decomposition of the Coulomb interaction, leading to the action

$$S = \int dt\, d\mathbf{r} \Bigg\{ \psi^+ i \partial_t \psi + \frac{1}{2m} \psi^+ [i\nabla - \mathbf{A}_a S^z]^2 \psi(\mathbf{r}) - \rho_s(\mathbf{r}) V_H^{(1)}(\mathbf{r}) - \rho^z(\mathbf{r}) V_H^{(2)}(\mathbf{r}) - V_{\mathrm{ex}} \mathbf{m} \cdot \psi_\sigma^+(\mathbf{r}) \mathbf{S}_{\sigma\sigma'} \psi_{\sigma'}(\mathbf{r}) + \frac{1}{2} V_{\mathrm{ex}} \mathbf{m}^2 + \frac{1}{2} \int d\mathbf{r}'\, n(\mathbf{r}) V_s(\mathbf{r} - \mathbf{r}') n(\mathbf{r}') + \frac{1}{2} \int d\mathbf{r}'\, m_z(\mathbf{r}) V_a(\mathbf{r} - \mathbf{r}') m_z(\mathbf{r}') \Bigg\} \qquad (1)$$

In Eq. (1) ψ^+, ψ are fermionic fields for the electrons. The symmetric and antisymmetric densities are $\rho_s \equiv \psi_\sigma^+ \psi_\sigma$ and $\rho^z \equiv \psi_\sigma^+ S_{\sigma\sigma'}^z \psi_\sigma'$, with S^i being the Pauli matrix for the ith component of the isospin ($i = x, y, z$). The fields $n(\mathbf{r}), \mathbf{m}(\mathbf{r})$ are auxiliary Hubbard-Stratanovich fields

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describing symmetric and antisymmetric densities. We are interested in the response of the system to a weak antisymmetric vector potential $\mathbf{A}_a$, which is thus included in the action (a factor of $\frac{e}{c}$ is absorbed in $\mathbf{A}_a$). In momentum representation, $V_s(\mathbf{q}) = \frac{2\pi e^2}{q}$ and $V_a(\mathbf{q}) = 2\pi e^2 d$ (for small $\mathbf{q}$). For simplicity, we assume here that the true electron spin is fully aligned due to the Stoner instability and can be ignored (see, however, [10]).

In momentum representation, for small $\mathbf{q}$, the symmetric Hartree potential is $V_H^{(1)} = V_s(\mathbf{q})n(\mathbf{q})$ while the antisymmetric is $V_H^{(2)} = V_a(\mathbf{q})m_z(\mathbf{q})$. The Fock potential V_{ex} is approximated in Eq. (1) to be local, thereby neglecting the exchange contribution to the gradient terms which contribute to the isospin stiffness. We comment on the actual value of V_{ex} and on consequences of its nonzero range below. In the system's response to $\mathbf{A}_a$ the symmetric field $n(\mathbf{r})$ does not play any role and we omit it from following expressions.

For fixed values of $\mathbf{m}$ and n, the action (1) describes noninteracting electrons under the influence of a space and time dependent scalar potential $V_H^{(1)}$, vector potential $\mathbf{A}_a$, and Zeeman field $V_H^{(2)}\hat{z} + V_{\text{ex}}\mathbf{m}$. In an $x - y$ ordered state, the saddle point for the bosonic fields is $n(\mathbf{r}) = m_z(\mathbf{r}) = 0$, and $|\mathbf{m}(\mathbf{r})| = \mathcal{M}$, a nonzero constant. Conventional approximation schemes [HF, random phase approximation (RPA)] do not reliably obtain $\mathcal{M}$. Here we first assume full polarization ($\mathcal{M} = n$, as predicted by HF), and later discuss the case of partial polarization.

Because of an assumed lack of interlayer tunneling, the action (1) possesses a U(1) symmetry [3]. Thus, in equilibrium the system picks an arbitrary direction for $\mathbf{m}$. We write $\mathbf{m} = \mathcal{M}\{\cos[\varphi(\mathbf{r})]\hat{\mathbf{x}} + \sin[\varphi(\mathbf{r})]\hat{\mathbf{y}}\} + m_z\hat{\mathbf{z}}$, where φ is the angle between the planar component of the magnetization and the $\hat{x}$ axis, and we expect φ to be constant in the ground state and slowly varying in low-energy excitations. The energy cost of a deviation from the equilibrium magnetization is then expressed in terms of φ and m_z and should vanish for a uniform shift in $\varphi(\mathbf{r})$.

We now integrate over the fermionic fields and expand the familiar tr log{} term to second order in φ and m_z. Within RPA the expansion is given in terms of the response functions $\chi_i \equiv -\langle\rho^i\rho^i\rangle$ and $\chi_o \equiv -\langle\rho^z\rho^y\rangle$. The effect of a slowly varying $\mathbf{A}_a$ on the χ's can be separated out by means of a Gorkov approximation, where $\mathbf{A}_a$ is approximated not to vary in the range of $\mathbf{r} - \mathbf{r}'$ and $t - t'$ in which the response function is appreciable [11]. The effect of $\mathbf{A}_a$ is then incorporated by the "minimal coupling" prescription $i\nabla\varphi \to (i\nabla\varphi - \mathbf{A}_a)$, and the response functions are calculated for $\mathbf{A}_a = 0$. The RPA action is then (omitting the zeroth order term)

$$S_{\text{RPA}} \approx \frac{1}{2}\int d\omega \int d\mathbf{r}\Big\{\rho_s|i\nabla\varphi - \mathbf{A}_a|^2 + \frac{e^2}{\Gamma}m_z^2 + 2\chi_o V_{\text{ex}}\mathcal{M} \times (V_{\text{ex}} + 2\pi e^2 d)m_z\varphi\Big\}, \quad (2)$$

where [12]

$$\rho_s \equiv -\lim_{\mathbf{q},\omega\to 0} q^{-2}(1 + \chi_y V_{\text{ex}})V_{\text{ex}}\mathcal{M}^2, \quad (3)$$

$$\frac{1}{\Gamma} \equiv -\lim_{\mathbf{q},\omega\to 0}[1 + \chi_z(2\pi e^2 d + V_{\text{ex}})](2\pi e^2 d + V_{\text{ex}}). \quad (4)$$

The response functions χ_o, χ_z, χ_y are response functions of noninteracting electrons in a Zeeman field $V_{\text{ex}}\mathcal{M}\hat{x}$. For small $\mathbf{q}, \omega$,

$$\chi_z = \chi_y = -\frac{1}{2}\left[\frac{\mathcal{M}}{\Delta - \omega - \mathcal{D}q^2} + \frac{\mathcal{M}}{\Delta + \omega - \mathcal{D}q^2}\right], \quad (5)$$
$$\chi_o = -\frac{1}{2}\left[\frac{i\mathcal{M}}{\Delta - \omega - \mathcal{D}q^2} - \frac{i\mathcal{M}}{\Delta + \omega - \mathcal{D}q^2}\right],$$

where $\Delta \equiv \mathcal{M}V_{\text{ex}}$ is the energy cost for flipping a spin, and the value of $\mathcal{D}$ is discussed below. The U(1) invariance of the problem is the reason for $1 + \chi_y V_{\text{ex}}$ being $\mathcal{O}(q^2)$. The equation of motion for m_z, derived from (2), is the Josephson-type relation $\dot{\varphi} = 2\pi e^2 d m_z$.

The integral over m_z can now be carried out, resulting in an action in terms of φ and $\mathbf{A}$ only, which is more transparent in space and time representation:

$$\int dt \int d\mathbf{r}\, \frac{\dot{\varphi}^2}{2}\left[\frac{1}{2\pi e^2 d} + \frac{1}{V_{\text{ex}}}\right] + \frac{\mathcal{M}\mathcal{D}}{2}|(i\nabla - \mathbf{A}_a)\varphi|^2. \quad (6)$$

Equation (6) is the action of a two-dimensional superfluid, with $\mathcal{M}\mathcal{D}$ being the superfluid "spin stiffness." *If $\mathcal{M}\mathcal{D} > 0$, the bilayer system responds to the vector potential $\mathbf{A}_a$ as a superfluid, and an antisymmetric current flows without dissipation.* Equation (6) reveals the existence of a longitudinal Goldstone mode that carries antisymmetric density and satisfies the dispersion relation

$$\omega^2 = \mathcal{M}\mathcal{D}\left[\frac{1}{2\pi e^2 d} + \frac{1}{V_{\text{ex}}}\right]^{-1} q^2. \quad (7)$$

This Goldstone mode corresponds to the spin wave for the pseudospin ferromagnet (much like the spin wave of the real ferromagnet [5,13]) modified by the presence of easy-plane anisotropy.

Within RPA, the response functions χ_y, χ_z are

$$\chi_y(\mathbf{q},\omega) = \chi_z(\mathbf{q},\omega) = \sum_{\alpha\beta}\frac{1}{2}|\langle\alpha|\rho_{\mathbf{q}}|\beta\rangle|^2 \times \left\{\frac{f(\epsilon_\alpha + \Delta) - f(\epsilon_\beta)}{\omega + \Delta + \epsilon_\alpha - \epsilon_\beta + i\eta} + \frac{f(\epsilon_\alpha) - f(\epsilon_\beta + \Delta)}{\omega - \Delta + \epsilon_\alpha - \epsilon_\beta + i\eta}\right\}, \quad (8)$$

where $|\alpha\rangle, |\beta\rangle$ are single particle eigenstates of the spin-independent noninteracting Hamiltonian, $\epsilon_\alpha, \epsilon_\beta$ are the corresponding single particle energies, $\rho_{\mathbf{q}}$ is the density operator, and $f(\epsilon)$ is the Fermi function.

Setting $\omega = 0$ and expanding to second order in q, we find that for a clean system with full isospin polarization ($2\Delta > \mu$, μ being the chemical potential)

$$\mathcal{M}\mathcal{D} = \int_{k<k_F} d\mathbf{k}\left[\frac{1}{m} - \frac{k^2}{2m^2\Delta}\right] = \frac{n}{m}\left(1 - \frac{\mu}{2\Delta}\right). \tag{9}$$

This energy cost is the sum of single particle energies of eigenstates $|\mathbf{k}\rangle$ of electrons in a Zeeman magnetic field that precesses in space in a constant rate $\nabla\varphi$ and is composed of one part $\frac{(\nabla\varphi)^2}{2m}$ originating from the antisymmetric current induced by the precession of the field, and a second part, $-\frac{(\mathbf{k}\cdot\nabla\varphi)^2}{2m^2\Delta}$, which reflects the slowing down of the symmetric motion due to the field precession. There is no Galilean invariance for antisymmetric currents so $\mathcal{M}\mathcal{D} \neq n/m$.

The disorder potential can be separated into symmetric and antisymmetric parts. The symmetric part affects $\mathcal{D}$ much like nonmagnetic disorder does in a conventional superconductor. For weak symmetric disorder ($k_F l \gg 1$, and hence $\Delta\tau \gg 1$), the disorder-averaged matrix elements in (8) are

$$|\langle\alpha|\rho_{\mathbf{q}}|\beta\rangle|^2 = \frac{1}{\nu(\bar\epsilon)}\frac{D(\bar\epsilon)q^2}{[D(\bar\epsilon)q^2]^2 + (\epsilon_\alpha - \epsilon_\beta)^2}, \tag{10}$$

where $\bar\epsilon = \frac{1}{2}(\epsilon_\alpha + \epsilon_\beta)$, D is the diffusion constant, ν is the density of states, and $|\epsilon_\alpha - \epsilon_\beta| < \frac{1}{\tau}$. Substituting in (8) and paying attention to the dependences of ν and D on $\bar\epsilon$ we find that the effect of symmetric disorder on the spin stiffness (9) is of order $1/\Delta\tau$.

Antisymmetric disorder modifies the capacitive energy term in (1) to be $2\pi e^2 d\int d\mathbf{r}[m_z(\mathbf{r}) - m_{z,\mathrm{dis}}(\mathbf{r})]^2$ with random $m_{z,\mathrm{dis}}$. As is known from studies of, e.g., Josephson junction arrays, such a randomization in the equilibrium distribution of m_z reduces the superfluid density and can, if strong enough, induce vortex-antivortex pairs destroying the superfluidity even at zero temperature. Here, since there are gapless Fermi surface excitations even in the superfluid, the resultant disordered phase may possibly be a normal Fermi liquid with no long-range interlayer phase coherence.

Realistically, the disorder potential is made of comparable symmetric and antisymmetric components. For weak disorder, then, antisymmetric currents flow without dissipation, although the superfluid density is suppressed. Strong disorder eventually destroys the superfluidity.

The superfluid spin stiffness $\mathcal{M}\mathcal{D}$ is also suppressed by finite temperature. Just as in an ordinary superconductor, its temperature dependence originates both from the Fermi functions in (8) and from thermal fluctuations of vortex-antivortex pairs in $\varphi(\mathbf{r})$. The spin stiffness, and with it long-range order and antisymmetric dissipationless transport, disappear entirely above a Kosterlitz-Thouless (KT) transition temperature, whose precise value depends on both effects.

An experimental probe of superfluidity of antisymmetric currents is the transresistance, or drag resistance, denoted by ρ_D. In a drag measurement a current I_1 is driven in one of the layers, while no current is allowed to pass through the second layer ($I_2 = 0$) which develops a voltage V_2. Then, $\rho_D \equiv -V_2/I_1$. For two identical layers, ρ_D is the difference between the symmetric and antisymmetric resistances. In our case the latter vanishes. Thus, *the transresistance equals the symmetric one, and the voltages on the two layers should be equal in magnitude and direction.* Since the superfluidity disappears at the KT transition temperature, ρ_D would go *down* with increasing temperature. Note that for weakly coupled Fermi liquid bilayer systems ρ_D is opposite in sign to the intralayer resistance, and its magnitude *increases* with temperature. If the superfluid mode is lost due to disorder, the antisymmetric resistance becomes appreciable, and the sign of ρ_D presumably becomes opposite to that of the intralayer resistance.

The excitation of the sound mode (7) is another experimental probe. In the absence of isospin ferromagnetism, a double layer system has an antisymmetric acoustic plasmon mode, which is overdamped by disorder as $q \to 0$ [14]. Here, however, the sound mode (7) is an underdamped Goldstone mode. A density sweep experiment through the transition will therefore exhibit a sharp mode at low density which will get overdamped (at long wavelengths) above the transition density. Another distinction between these two collective modes is their behavior when $d \to 0$. In that limit the Goldstone mode will have a long wavelength quadratic q^2 dispersion, since the U(1) isospin symmetry changes into an SU(2), whereas the normal acoustic plasmon mode tends toward the single particle dispersion $v_F q$.

So far we have taken the exchange Fock potential to be local, and employed RPA. The U(1) symmetry of the approximate actions (2)and (6) is exact. However, other features of our analysis are not, of which we expect two to be most important. First, as a consequence of its finite range, the Fock potential renormalizes the dispersion relation of the electrons $\epsilon(k)$. The sum (8) should then be evaluated with the renormalized energy dispersion, leading to the replacements $\frac{1}{m} \to \frac{\partial^2\epsilon}{\partial k^2}$ and $\frac{k}{m} \to \frac{\partial\epsilon}{\partial k}$ in the integral in (9), and affecting the spin stiffness.

Second, it is conceivable that the Stoner phase is only partially isospin polarized, in contrast to the HF prediction of full polarization. Interestingly, for such a state, and in the absence of disorder and electron-electron interaction, the q^2 term in the sum (8) vanishes (i.e., $\mathcal{D} = 0$), due to the constant density of states. Spin stiffness is then induced by the deviation of ν from a constant, caused by the renormalization of the energy dispersion by interaction. Similarly, in the presence of symmetric disorder, $\mathcal{D} \propto [(D\nu)'_{(\mu+\Delta/2)} - (D\nu)'_{(\mu-\Delta/2)}]$, where a prime denotes differentiation with respect to energy. Again, the energy dispersion must deviate from parabolic for $\mathcal{D}$ to be nonzero.

The physics of the isospin ferromagnet at filling factor $\nu = 1$ in the QHE regime is quite different from that at $B = 0$. In the presence of interlayer phase coherence, the finite isospin stiffness leads to an energy gap for symmetric excitations and a QHE plateau [1,4,5].

Because of the energy gap for symmetric excitations, the fermions can be reliably integrated out [1,4,5] to yield a Euclidean action which is a functional of the unit vector $\hat{\sigma}(\mathbf{r}) \equiv \mathbf{m}(\mathbf{r})/n(\mathbf{r})$. The Hamiltonian density is

$$H = \frac{1}{2}\bar{\rho}\,\partial_\mu\sigma^\nu\partial_\mu\sigma^\nu + \frac{e^2}{2\Gamma}[n(\mathbf{r})\sigma^z]^2 + V_s(\mathbf{r})\delta n(\mathbf{r}) + V_a(\mathbf{r})[n_0 + \delta n(\mathbf{r})]\sigma^z(\mathbf{r}). \quad (11)$$

Γ is the double layer capacitance per unit area (including Hartree and exchange contributions), $V_{s,a}$ are the symmetric and antisymmetric parts of the disorder potential, and $\bar{\rho} \sim 1$ K is the exchange-induced spin stiffness [4,5].

The quantization of the Hall conductivity imposes a constraint relating the symmetric fermion density to the topological (Pontryagin) density of the field σ [1,4,5]:

$$\delta n(\mathbf{r}) = \left(\frac{h}{e^2}\,\sigma_{xy}\right)\frac{1}{8\pi}\,\epsilon^{\mu\nu}\epsilon_{abc}\sigma^a\partial_\mu\sigma^b\partial_\nu\sigma^c. \quad (12)$$

Taking advantage of the easy-plane anisotropy and noting that the *XY* phase angle field φ contains vortex singularities, we integrate out the massive σ^z fluctuations and find that the lack of time-reversal symmetry causes the disorder potential to generate a gauge field yielding, in the high temperature classical limit, *a 2D XY model with random Dzyaloshinskii-Moriya interaction*

$$\int d\mathbf{r}\,\frac{\bar{\rho}}{2}\,|\nabla\varphi + \mathbf{a}|^2 + \sum_j[\lambda V_a(\mathbf{R}_j) - V_s(\mathbf{R}_j)Q_j]M_j\,, \quad (13)$$

where $Q_j = \pm 1$ is the vorticity of the jth vortex ("meron" [4,5]), $M_j = \pm 1$ is a flavor index indicating the sign of σ^z in the vortex core, λ is a nonuniversal constant related to the core size, and we have dropped various irrelevant terms (e.g., a random contribution to $\bar{\rho}$). The gauge field is $\mathbf{a} \equiv \frac{\Gamma V_a}{4\pi\bar{\rho}e^2}\mathbf{J}_+$, where $\mathbf{J}_+ = -\frac{h}{e^2}\sigma_{xy}\epsilon^{\mu\nu}\partial_\nu V_s$ is proportional to the symmetric Hall current. The ΓV_a term is the local density imbalance. If there is a Hall current flowing when there is a density imbalance, then more current is flowing in one layer than the other and the superfluid mode [4,5] $\mathbf{J}_- \sim \bar{\rho}(\nabla\varphi + \mathbf{a})$ is necessarily excited. This is the physical interpretation of the gauge potential $\mathbf{a}$ which causes these currents to flow. The field ϕ in (13) contains both singular and smooth parts, and thus the first term in (13) mediates a logarithmic interaction between the merons. In the SU(2) symmetric quantum Hall ferromagnet, studied by Green [13], a symmetric disorder potential was mapped onto a random vector potential coupled to a two component order parameter (corresponding, in our problem, to σ_y, σ_z). In the present case, fluctuations in σ_z are already integrated out, and a combination of both V_s and V_a is mapped onto a random vector potential coupled to the phase field φ.

The gauge field $\mathbf{a}$ is random with a finite correlation length. Ensemble averaging over a closed contour $\partial\gamma$ of perimeter L gives $\langle\oint_{\partial\gamma}\mathbf{a}\cdot d\mathbf{r}\rangle = 0$, and $\langle[\oint_{\partial\gamma}\mathbf{a}\cdot d\mathbf{r}]^2\rangle \sim L^\theta$ with $\theta = 1$. This is the gauge glass model for which it is known that the KT transition is destroyed in the limit of strong disorder [15]. For weak disorder the phase diagram has proven difficult to determine [16], but it is likely that the KT transition survives. Note that in order to have an isolated flux quantum through a single plaquette, the vector potential would have to fall off like $1/r$ and it follows that the random potential $V_{s,a}$ would have to diverge.

To conclude, our results call for an experimental search via light scattering and drag transport for the $B = 0$ phase coherent state in large r_s bilayer hole systems, and for a study of how superfluidity in this phase and its analog at $\nu = 1$ is affected by disorder.

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