Fractionalization in the Cuprates: Detecting the Topological Order

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(Received 7 July 2000)

The precise theoretical characterization of a fractionalized phase in spatial dimensions higher than one is through the concept of "topological order." We describe a physical effect that is a robust and a direct consequence of this hidden order that should enable a precise experimental characterization of fractionalized phases. In particular, we propose specific "smoking-gun" experiments to unambiguously settle the issue of electron fractionalization in the underdoped cuprates.

DOI: 10.1103/PhysRevLett.86.292 PACS numbers: 74.20.Mn, 74.72.–h

Does the electron splinter apart (i.e., fractionalize) in the cuprate high-T_c materials? This question has been the subject of heated debate for some years, and is apparently far from being settled. Phenomenologically, electron fractionalizion [1–4] is very appealing, primarily because it provides a simple explanation of the superconductivity. Upon fractionalization the charge of the electron is liberated from its Fermi statistics, thereby allowing the electron charge to *directly* condense leading to superconductivity without invoking ideas of pairing. Further, several other unusual properties of the cuprates find a natural explanation in terms of the fractionalization idea, most notably the angle-resolved photoemission results [4].

Despite these attractive features, there has been considerable difficulty in constructing an acceptable theory of fractionalization for the cuprates. This is partly due to inadequacies in our theoretical understanding, but more problematic has been the lack of clear experimental signatures indicative of electron fractionalization. In this paper, building on our recent theoretical understanding of fractionalization using a Z_2 gauge theory formulation [5], we overcome this difficulty. In particular, we predict a novel physical effect that is a robust property of fractionalized phases—this makes possible a direct and unambiguous experimental test of whether the electron splinters apart in the cuprates.

Fractionalization theory.—In our recent work [5] we demonstrated that a general class of strongly interacting electron models could be recast in the form of a Z_2 gauge theory, which then enabled us to provide a reliable discussion of issues of electron fractionalization. In particular, we demonstrated the possibility of obtaining fractionalized phases in two or higher spatial dimensions. In such a phase, the electron splits into two independent excitations—the spin of the electron is carried by a neutral fermionic excitation (the "spinon") and the charge is carried by a bosonic excitation (the "chargon") [6]. There is a third distinct excitation, namely, the flux of the Z_2 gauge field (dubbed the "vison"). The vison is gapped in the fractionalized phase. The Z_2 gauge theory approach is closely related to the ideas on vortex pairing [7] and other gauge theoretic formulations [8,9] of fractionalization.

The precise theoretical characterization of a fractionalized phase is in its "topological order" [5,10]—a concept that has been elucidated clearly in the context of the quantum Hall effect by Wen and co-workers [11]. A fractionalized phase in a manifold with a nontrivial topology has a ground state degeneracy which depends on the topology. A vison that is trapped in each "hole" in the manifold stays there forever, but does not affect the ground state energy in the thermodynamic limit. Consider for instance a cylinder. There are two states depending on whether or not a vison has threaded the cylinder. The inability of the trapped vison to escape from the cylinder is the hallmark of the fractionalized phase.

The topological order inherent in a fractionalized phase endows it with a tremendous amount of robustness to various "real-life" complications. For example, topological order survives [5] in the presence of weak amounts of disorder. It can also coexist with various other conventional broken symmetries, such as charge or spin ordering [4,5,7]. Thus, if the cuprates do show fractionalization, features like charge stripes or even antiferromagnetism are side effects (albeit interesting ones) and not directly related to the origin of the superconductivity.

It is also important to understand the role of a finite nonzero temperature on the topological order. Theoretically, the effect of finite temperature depends crucially [5] on the spatial dimension. In two spatial dimensions (2D), the visons are pointlike excitations, and are gapped in the fractionalized phase. But at any nonzero temperature, there will be a finite density of thermally excited visons, thereby destroying the topological order. In a three-dimensional fractionalized phase, however, the visons are looplike excitations, costing a finite energy per unit length. Consequently, at low temperatures large vison loops are absent and the topological order survives. As the temperature is increased there will eventually be a true phase transition where the vison loops unbind.

For quasi-two-dimensional layered materials such as the cuprates, two distinct fractionalized phases are possible. First, the fractionalization can occur independently in each layer with the different layers being "decoupled" from one another. In this case, vison loops proliferate *between* the

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layers, and the topological order exists only for visons which penetrate a layer. As in 2D, this order is destroyed by arbitrarily small temperature. Alternatively, with strong enough interlayer coupling it is possible that the interlayer vison loops are also expelled, resulting in a phase with full three-dimensional topological order that survives at low nonzero temperatures.

Detecting the topological order.—Armed with the theoretical understanding described above, we may pose a sharp and definite question about the cuprates. Quite generally, there are three qualitatively distinct possibilities for the behavior of the underdoped cuprates with regard to the phenomenon of fractionalization.

(i) Fractionalization and the associated topological order simply do not occur in the cuprates (see Fig. 1).

(ii) The topological order occurs independently in each two-dimensional layer, and hence strictly speaking, exists only at zero temperature (see Fig. 2).

(iii) The topological order is three dimensional, and survives up to a nonzero temperature (see Fig. 3).

Which one of these three possibilities is actually realized in the real materials? It is widely believed that the pseudogap line in the cuprates is only a crossover and not a true phase transition. This, and other phenomenological considerations (specifically the "incoherent" c-axis transport), probably disfavor 3D topological order as a serious possibility in the cuprates. Nevertheless, for conceptual purposes it will be very useful to begin by considering this case. Though this is distinguished from the other two cases by having a finite temperature phase transition, a *direct* experimental characterization of the 3D topological order would be preferable.

We now propose an experiment that is directly sensitive to the presence of topological order, presuming initially that scenario (iii) is realized. Imagine the following sequence of events.

(a) Start with an underdoped sample in a cylindrical geometry, with the axis of the cylinder perpendicular to the layers. In the presence of a magnetic field, cool into the superconducting phase such that exactly one $hc/2e$ flux quantum is trapped in the hole of the cylinder.

(b) Heat the sample to above T_c. *The magnetic flux penetrates into the sample, but the vison will still be trapped.* This is because of the finite temperature topological order in case (iii).

(c) Now turn off the magnetic field. The vison will still remain trapped. We have thereby prepared the sample above T_c with zero magnetic field in a state with a vison threading the cylinder.

(d) How do we tell that there is a vison trapped? The simple way is to cool the sample back down below T_c. The trapped vison still cannot escape but *must* nucleate an $hc/2e$ quantum of magnetic flux. This flux will be generated *spontaneously* and can be in either direction—thereby breaking the time reversal invariance achieved in (c).

An alternate experiment is to again repeat the sequence of events (a) to (d), but now work at a fixed very low temperature and move from the superconductor into the (underdoped) insulator, and back, by adiabatically tuning some parameter. Again, one should see a spontaneous $hc/2e$ flux generated if the ground state on the insulating side is fractionalized. This experiment is, of course, much more challenging. It has recently been demonstrated however that an electrostatic field [12] can be used to move across the superconductor-insulator phase boundary at low temperature, at least for very thin films. Another possibility is to use photodoping (or perhaps even pressure).

These experiments offer a direct and conceptually straightforward way to detect the presence of 3D topological order. In particular, if case (i) is what is actually realized, there will certainly be no spontaneous flux generated at the end of either experiment.

But what if the topological order is two dimensional [case (ii)], as seems more likely for the cuprates? In the

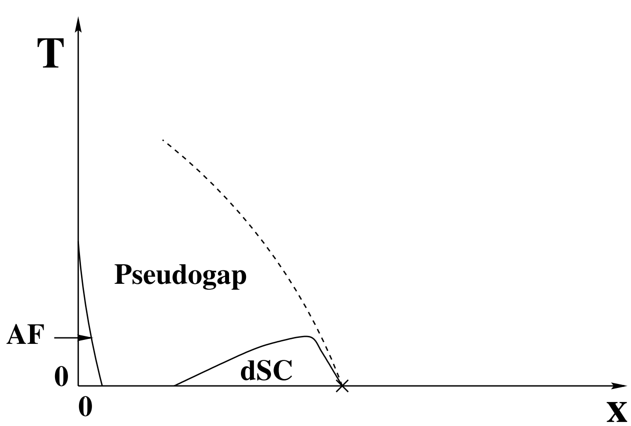

FIG. 1. One of the three possibilities for the underdoped cuprates with regard to electron fractionalization and the associated topological order: There simply is no topological order.

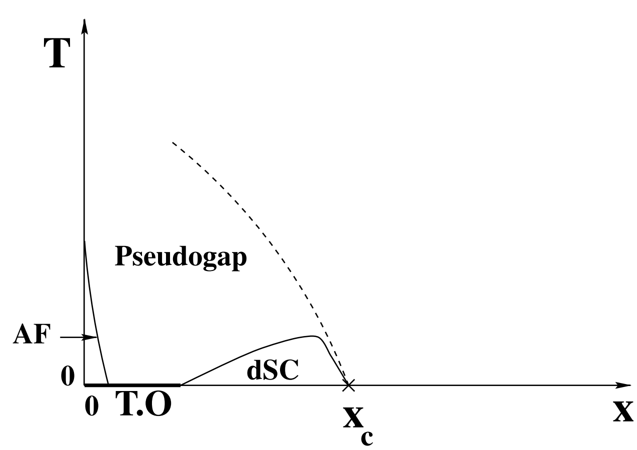

FIG. 2. Quasi-2D fractionalization: The topological order (T.O) is strictly present only at zero temperature. The dashed line describing the pseudogap crossover corresponds to the crossover to the $T = 0$ fractionalized phase.

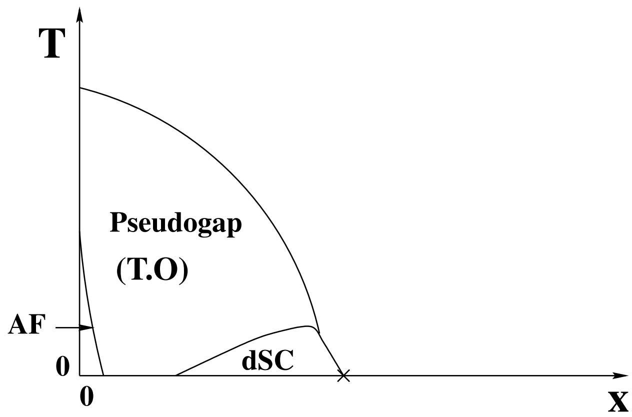

FIG. 3. 3D fractionalization: The topological order survives up to a finite nonzero temperature.

experiment performed by tuning across the superconductor-insulator phase boundary at *zero* temperature, a spontaneous flux is certainly expected. But at finite temperature the topological order is strictly speaking absent, and a trapped vison will ultimately escape at long times. As is usual in such cases, the result will then depend on how fast the experiment is performed. In step (b) above, the magnetic flux penetrates into the sample almost immediately but at low temperatures the vison will remain trapped for a much longer time of order $t_v \sim t_0 e^{E_0/k_B T}$. Here, E_0 is the energy cost for a vison in a single layer and t_0 is some microscopic time scale. Provided the time scale of the experiment is smaller than t_v, a spontaneous magnetic flux should be present at the end. But if the time scale of the experiment is significantly longer than t_v, there will be no flux and the presence of the ($T = 0$) topological order will be missed. It is therefore essential that the experiment be performed over time scales smaller than the vison decay time. To this end, it is clearly advantageous to heat the sample only slightly above T_c in step (b) above. Interlayer coupling will presumably enhance t_v, so that less anisotropic materials might also be preferable.

An advantage of these experiments is that they offer a *direct* and unambiguous route to detect the presence of topological order. With topological order, the spontaneous generation of flux should be insensitive to unavoidable material complications, such as impurities and other coexisting broken symmetries (stripes, etc). In that sense, this experiment gives a characterization of fractionalization that is as clear-cut as the Meissner effect is for superconductivity, or quantization of the Hall conductivity for the quantum Hall effect.

A number of other equally robust predictions can be made when the conditions in the above experiment are slightly modified. Specifically, if the initial magnetic flux trapped in step (a) is hc/e rather than $hc/2e$, there will be no trapped vison in step (b), hence there will be no spontaneous flux observed. More generally, if the initial flux is an odd multiple of $hc/2e$, a spontaneous flux of $hc/2e$ will be generated at the end, whereas with an even multiple of $hc/2e$ initially, there will be no spontaneous flux. This even/odd effect is a direct reflection of the Ising character of electron fractionalization above one dimension. Observation of this even/odd effect will be a strong evidence against mundane explanations of the effect such as the presence of unknown stray magnetic fields.

Another interesting geometry to consider, particularly if case (ii) is realized, is to repeat the experiment with the axis of the cylinder parallel to the layers. Then, the vison can escape by passing between adjacent layers, and no spontaneous flux will be generated (independent of the initial flux).

Once a vison is trapped in a topologically ordered phase, one can also imagine other more subtle physical effects that will distinguish it from the same sample with no trapped vison. For instance, thermal or even electrical conductance of a cylinder with a trapped vison will be different from that without the vison, since the interference contribution from paths of spinons or chargons that wind around the cylinder will be affected (extra minus sign for an odd winding if a vison is present). Detecting this effect will of course require "phase coherence" around the cylinder, which presumably occurs only at very low temperatures and in a small sample. In the cuprates with d-wave pairing, thermal transport may be preferable due to the presence of low energy spinon excitations which could transport heat. Another subtle effect is that the superconducting transition temperature should be slightly smaller if a vison is trapped.

Practical considerations.—The experiments proposed above are certainly challenging and would require a good deal of care. In tuning out of the superconducting phase either with temperature or by other means, it is important to make sure that the sample is definitely no longer superconducting. This could be done by monitoring the resistance simultaneously. But better still would be to check directly that no magnetic flux is still trapped after the external field is turned off in step (c).

It seems most likely that the topological order, if present at all in the cuprates, will be two dimensional in character. In that case, it is essential that the time scale of the experiment be smaller than the vison decay time t_v. Since t_v increases exponentially with the ratio of the vison gap to the temperature, the sample should be heated just above T_c in step (b). Moreover, if the vison gap is of the order of the pseudogap temperature T^* (e.g., measured in photoemission) as we proposed earlier [4], it would be preferable to work with *very* underdoped samples. This will maximize the vison decay time t_v, being exponentially large in the ratio T^* to T_c.

In this paper we have discussed a very general and robust physical effect that must be present in the underdoped cuprates if they exhibit electron fractionalization. The robustness of the effect is due to the topological order inherent in the fractionalized phase. Experimental confirmation of this effect would unambiguously establish the presence of fractionalization in the underdoped cuprates. Conversely, if the experiment fails to observe the effect when performed with sufficient care, it would establish the absence of fractionalization [13].

Electron fractionalization, and the associated topological order [14], may well be more widely prevalent in strongly interacting many-fermion systems than has been previously assumed. In particular, liquid and solid He-3, the two-dimensional electron gas at low density and a plethora of heavy fermion and organic materials all show a host of unusual phenomena which are poorly understood. Experiments along the lines of those proposed in this paper might enable a detection of the otherwise elusive topological order that may lurk in these systems. As in the cuprates, a proximate superconducting or superfluid phase would be required to detect — and manipulate — this "hidden" topological order.

We are grateful to Leon Balents, S. Sachdev, and Doug Scalapino for fruitful discussions. This research was generously supported by the NSF under Grants No. DMR-97-04005, No. DMR95-28578, and No. PHY94-07194.

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[13] Throughout this paper, we have focused on the kind of fractionalization that possibly occurs in two- or higher-dimensional systems. The fractionalization that occurs in one-dimensional systems is from a qualitatively different mechanism as discussed in Ref. [4]. If the cuprates do show this "one-dimensional" fractionalization due to the presence of stripelike structures, there will be no spontaneous flux at the end of the experiment.
[14] Theoretically, a variety of fractionalized phases with distinct topological orderings is possible (just like in the quantum Hall effect). In this paper, we have focused on the particular kind of fractionalization that is most relevant to the cuprates. For other materials, other distinct kinds of topological ordering leading to a different fractionalization pattern are possible.