Quantum confinement transition in a *d*-wave superconductor

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We study the nature of the zero-temperature phase transition between a *d*-wave superconductor and a Mott insulator in two dimensions. In this ''quantum confinement transition,'' spin and charge are confined to form the electron in the Mott insulator. Within a dual formulation, direct transitions from *d*-wave superconductors at half-filling to insulators with spin-Peierls (as well as other) order emerge naturally. The possibility of *striped superconductors* is also discussed within the dual formulation. The transition is described by nodal fermions and bosonic vortices, interacting via a long-ranged statistical interaction modeled by two coupled Chern-Simons gauge fields, and the critical properties of this model are discussed.

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I. INTRODUCTION

In recent years, due to remarkable experimental progress,[1] the cuprate superconductors have revealed a host of mysterious phases as their chemical doping is varied. Indeed, it would seem as though these materials exhibit many of the wide range of behaviors possible for low-dimensional, highly correlated electron systems. Centrally located within the phase diagram and adjacent to many of these puzzling regions is the *d*-wave superconductor. Beginning in this well-understood phase, one may develop theoretical descriptions of other nonsuperconducting phases. Of particular interest are the $T=0$ quantum phases, both in the very underdoped and heavily overdoped regimes. The schematic situation is shown in Fig. 1.

When describing a two-dimensional superconductor, topological defects in the Cooper pair wave function (BCS vortices) are of particular importance. Being bosonic, once they proliferate, they condense at $T=0$, destroying superconductivity.[2] In this way, a description of quantum phases with strong pairing correlations but lacking the phase coherence that is superconductivity emerges quite naturally as *vortex condensates*.[3] If the superconductor is *d* wave, there is the additional complication of low-energy quasiparticles. As recently emphasized,[4] there is a statistical interaction between these spin-carrying quasiparticles and the vortices, making the resulting theory strongly interacting.

Singlet-paired superconductors can be recast in a spin-charge-separated form:[5] the condensate carries all the charge but no spin, while the quasiparticles are electrically neutral with spin 1/2. Most other well-understood phases of electrons (such as the Fermi liquid) are spin-charge confined. It was recently argued[6] that many puzzling aspects of the cuprate phase diagram could be understood in terms of the fractionalization and confinement of electrons. In this approach, the regions containing the pseudogap (and superconducting) phase are characterized by the presence of spin-charge separation (electron fractionalization), and can be thought of as condensates of hc/e vortices,[7] while the heavily overdoped regions are spin-charge-confined, $hc/2e$ vortex condensates. Between the two, a quantum confinement transition might cause critical behavior over wide regions of the high-T_c phase diagram. Focusing on the zero-temperature transition out of the superconductor on the underdoped side, it must be characterized as either a condensation of hc/e or $hc/2e$ vortices. In the former case, a fractionalized insulator at zero temperature results. In the latter case, we have a more conventional, confined phase. The same can be said of the corresponding transition on the overdoped side. In both cases, the finite doping of the system presents significant theoretical challenges. A key feature of the cuprates is the close proximity between a *d*-wave superconducting phase and a Mott insulating phase. Here, we put aside the issue of finite doping and work at half-filling, looking at direct transitions between a *d*-wave superconductor and a confined Mott insulator. One microscopic model which has shown a quantum transition between these two phases at half-filling is the t-U-W model, studied numerically by Assad and co-workers.[8] Here, we work instead with a dual Landau-Ginzberg model which enables us to access theoretically this transition by approaching from the superconducting phase. Our hope is to capture some of the physics of the confinement transition in the cuprates, at whichever doping it occurs (see Fig. 1). In particular, we seek to answer two broad questions regarding the nature of such a transition.

First, in terms of phenomenology, what sort of states might we find when separate spin- and charge-carrying excitations are confined to form electrons? As we shall see, a remarkable feature of superconductivity with one electron per unit cell is that in the dual theory, the vortices are fully

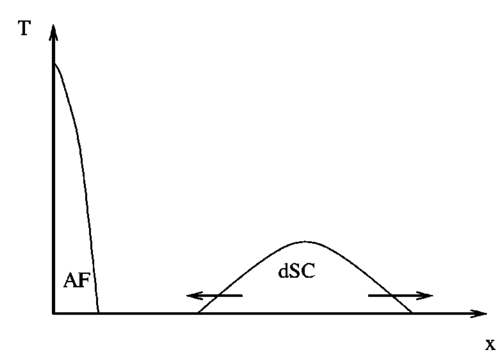

FIG. 1. Schematic phase diagram for the high-T_c cuprates.

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frustrated. When the vortices proliferate and condense, this frustration leads to the existence of multiple vortex condensates which break spatial symmetries. In particular, we find vortex condensates which destroy superconductivity (and, at half-filling, describe Mott insulators) as well as vortex condensates which preserve superconductivity. As a consequence of the vortex frustration, we find direct transitions from superconducting states to insulating states which spontaneously break rotational and/or translational symmetries, as well as the existence of superconducting states which have nontrivial spatial structure. Although we work at exactly one electron per unit cell, where the vortex theory is *fully* frustrated, it is hoped that even away from half-filling the qualitative features of our results will remain valid, in particular, the tendency toward spatial modulation near half-filling. In general, we hope that our explorations of frustrated vortex systems can yield insights into quantum phases of electrons which are complicated by the presence of competing interactions.

Second, as a specific example, we look at the critical properties of the the confinement transition between a spatially modulated d-wave superconductor and a Mott insulator with the same broken translational symmetry. Characterized by the presence of long-ranged statistical interactions, which affect the confinement of spin and charge, this quantum critical point should have interesting universal properties. Within a special region of parameter space, we explore this transition analytically using renormalization group (RG) methods.

Before we begin to address these questions, we first lay out the basics of the model under consideration. This model was introduced in Ref. 4 and many of its justifications and consequences can be found therein. Here, we provide only a whirlwind tour of its derivation and usefulness.

II. MODEL: Z_2 GAUGE THEORY

We begin by formally writing the electron creation operator as a product of *two* operators, one of which carries the spin of the electron and the other the charge. These operators are defined with the singlet-paired superconductor in mind. If we write the Cooper pair creation operator as $e^{i\varphi_r}$, we construct our spinless charge e boson (called a ''chargon'') from the Cooper pair as

$$b_r^\dagger = s_r e^{i\varphi_r/2} \equiv e^{i\phi_r}, \quad s_r = \pm 1. \tag{1}$$

The chargon is ''half a Cooper pair'' in the sense that the *square* of $b_r^\dagger$ creates a Cooper pair. The neutral spin-1/2 particle (called a ''spinon'') is obtained by removing the charge from the electron:

$$f_{r\alpha}^\dagger = b_r c_{r\alpha}^\dagger. \tag{2}$$

As we shall see shortly, this spinon can be thought of as a neutralized BCS quasiparticle. With these definitions, we may perform a change of variables in a suitable Hamiltonian describing electrons and Cooper pairs, resulting in a theory of chargons and spinons. However, the Hilbert space of chargons and spinons is much larger than that of electrons; for instance, the state with a single spinon but no chargons can be written down, but this state is unphysical and should be removed from the working Hilbert space. In other words, we may make this change of variables only if we additionally impose a *constraint* that the sum of the number of chargons, N_r (canonically conjugate to the chargon phase, $[\phi_r, N_{r'}] = i\delta_{rr'}$), and the number of spinons, $\rho_r = f_{r\alpha}^\dagger f_{r\alpha}$, on each site is an even integer:

$$(-1)^{N_r + \rho_r} = 1. \tag{3}$$

This constraint can be implemented within a Euclidean path integral representation, resulting in a theory of spinons and chargons coupled to a Z_2 gauge field.[4] It should be noted that the constraint used here is not the same as Gutzwiller projection, and does not disallow doubly occupied sites.

For an odd number of electrons per unit cell and d-wave pairing correlations, the appropriate action in the Z_2 gauge theory is

$$S = S_c + S_s + S_B, \tag{4}$$

$$S_c = -t_c \sum_{\langle ij \rangle} \sigma_{ij} (b_i^* b_j + \text{H.c.}), \tag{5}$$

$$S_s = -\sum_{\langle ij \rangle} \sigma_{ij} (t_{ij}^s \overline{f}_i f_j + t_{ij}^\Delta f_{i\uparrow} f_{j\downarrow} + \text{c.c.}) - \sum_i \overline{f}_i f_i, \tag{6}$$

$$S_B = -i \frac{\pi}{2} \sum_{i,j=i-\hat{\tau}} (1 - \sigma_{ij}), \tag{7}$$

where i and j label sites on a cubic space-time lattice. The Ising gauge field minimally coupled to the chargons and spinons, σ_{ij}, can take values ± 1, and S_B is a Berry's phase term.

One may arrive at this action by making the above-mentioned change of variables in a Hubbard-type Hamiltonian, as described in Ref. 4. Alternatively, this model can be taken as a starting point for describing systems with local singlet pairing correlations as well as Mott insulating tendencies. To exhibit the reasonableness of this model, consider the limits of infinite and vanishing t_c. For $t_c \to \infty$, the bosonic chargons will condense and the Z_2 gauge field will become frozen with $\sigma_{ij} = 1$, which frees the spinons. This phase is simply the d-wave superconductor. The action reduces to $S = S_s$, which is just the Bogoliubov–de Gennes action, with the spinons becoming the BCS d-wave quasiparticles. In the opposite limit $t_c \to 0$, the chargons are gapped into an insulating state. At $t_c = 0$, the chargons may be trivially integrated out. The remaining action is just $S = S_s + S_B$. It is shown in Ref. 4 that the partition function for this remaining spin theory is formally equivalent to that of the Heisenberg antiferromagnetic spin model. Therefore, we see the attractiveness of this model for the cuprate system, which also exhibits both superconductivity and antiferromagnetism. Many other additional properties of this action between these two limits are elucidated in Ref. 4, in particular, the presence of both spin-charge-*confined* and -*deconfined* phases.[9]

The charge sector in Eq. (4) is described in terms of the bosonic chargons, minimally coupled to a Z_2 gauge field. In two spatial dimensions, vortices in the boson many-body wave function are point like. This allows for a particularly elegant dual description where the vortex rather than the chargon is the fundamental degree of freedom. In this duality, the condensate of chargons (the superconductor) is the vacuum of vortices; the condensate of vortices is an electronic insulator, where the chargons are gapped. Within the vortex theory, the superconductor is trivial (being just the vacuum) and is therefore a good place to plant our feet. From this vantage, we look out of the superconductor at the neighboring insulating phases. The duality transformation, on the lattice, in the presence of the Z_2 gauge field, has been explicitly implemented in Ref. 4. The full resulting action at half-filling is

$$S=S_s+S_v+S_a+S_{CS}, \tag{8}$$

$$S_v=-t_v\sum_{\langle ij\rangle}\mu_{ij}\cos\left(\theta_i-\theta_j+\frac{a_{ij}}{2}\right), \tag{9}$$

$$S_a=\frac{\kappa}{8\pi^2}\sum_{\Box}|\Delta\times a_{ij}-2\pi\hat{\tau}|^2, \tag{10}$$

$$S_{CS}=\sum i\frac{\pi}{4}\left(1-\prod_{\Box}\sigma\right)(1-\mu_{ij}). \tag{11}$$

The spinon action S_s is unchanged. Here, $e^{i\theta_i}$ creates an $hc/2e$ vortex and the flux of the U(1) gauge field, a_{ij}, is the total electrical current. In particular, a flux of 2π through a spatial plaquette represents a charge of e. The terms S_v and S_a together form the usual dual-vortex representation for charge-$2e$ Cooper pairs except that here the vortices are minimally coupled to the additional (Z_2) gauge field $\mu_{ij}=\pm 1$. The BCS vortex and the spinon are relative semions; upon circling a vortex, the spinon wave function picks up a minus sign. The term S_{CS} is the Z_2 analog of a Chern-Simons term for the two Z_2 gauge fields and mediates this statistical vortex-spinon interaction. The spinons "see" a Z_2 flux $\Pi_{\Box}\sigma=-1$ attached to each $hc/2e$ vortex, while the vortices see a flux of $\Pi_{\Box}\mu=(-1)^{J_f}$. This flux attachment may be familiar to many in the context of the quantum Hall effect, where the gauge fields involved are for the U(1) group. Because of the anomalous "ff" terms in the action, spinon number is not conserved, and the usual Chern-Simons term cannot be used.

In the superconducting state, we are in the vacuum of vortices. The spinons see no flux and are free to propagate independently of the chargons. However, when single vortices condense, the long-range statistical interaction between the BCS vortex and the spinon drives spin-charge confinement. In the language of Ref. 4, the condensation of $hc/2e$ vortices is accompanied by a condensation of the visons (vortices in the Ising field, σ), leading to a confined phase of electrons.

We wish here to explore in some detail the nature of this confinement transition, where the freely propagating spin and charge excitations are "glued together" to form the electron. Aspects and implications of this quantum critical point pertaining to the high-T_c phase diagram have been introduced in Ref. 6. First, we will use Landau theory to find phases related to the d-wave superconductor by a second-order phase transition. Then, we will consider a special case where we recover a U(1) symmetry for the spinons and will use quantum field theory methods to extract some analytic critical properties of the transition between deconfined and confined phases.

III. DUAL-VORTEX THEORY AT HALF-FILLING

Concentrating on the vortices for the time being and working at half-filling, the dual theory for the charge sector becomes

$$\mathcal{L}_v=-t_v\cos\left(\theta_i-\theta_j+\frac{a_{ij}}{2}\right)+\frac{1}{2}|\nabla\times\mathbf{a}-2\pi\hat{\tau}|^2. \tag{12}$$

To obtain a low-energy effective theory, we work with a "soft-spin" model where the vortex creation operator $e^{i\theta}$ is replaced by a complex field Φ. In the interest of exploring the simplest case we set the charge per unit cell to be exactly e. In the dual theory, this corresponds to setting

$$\langle(\nabla\times\mathbf{a})_{\hat{\tau}}\rangle=2\pi. \tag{13}$$

In this section, we drop fluctuations of the gauge field $\mathbf{a}$, and consider a Landau mean-field approach. This is justified when the on-site repulsion between the electrons, U, is large. The vortices now see exactly $(\vec{\nabla}\times\vec{a}/2)_{\hat{\tau}}=\pi$ flux per spatial plaquette, and we are left with the two-dimensional (2D) fully frustrated quantum XY model:

$$S=\int d\tau\Bigg\{\sum_{\vec{r}}|\partial_\tau\Phi_{\vec{r}}|^2-\sum_{\langle\vec{r},\vec{r}'\rangle}t_{\vec{r}\vec{r}'}(\Phi^*_{\vec{r}}\Phi_{\vec{r}'}+\text{c.c.})+\sum_{\vec{r}}[m^2|\Phi_{\vec{r}}|^2+u(|\Phi_{\vec{r}}|^2)^2]\Bigg\}, \tag{14}$$

where $\vec{r}$ labels sites on the 2D square lattice dual to the original electron lattice and the sign of $t_{r,r'}$ around a plaquette is -1. The sites of the dual lattice are at the centers of the plaquettes of the original lattice, and in units of the lattice constant $(a=1)$, $\vec{r}=(x,y)$ with x and y integers.

We proceed, following closely the work of others on the fully frustrated quantum Ising model,[10] by choosing the gauge (to be used in the remainder of this paper) seen in Fig. 2. We may diagonalize the kinetic piece of this action to find two low-energy modes, residing at $(k_x,k_y)=(0,0)$ and $(\pi,0)$, respectively. In real space, these (unnormalized) eigenvectors are

$$\chi^0_{\vec{r}}=(1+\sqrt{2})-e^{i\pi y}, \tag{15}$$

$$\chi^\pi_{\vec{r}}=e^{i\pi x}[(1+\sqrt{2})+e^{i\pi y}]\quad (x,y\ \text{integers}). \tag{16}$$

For the purpose of characterizing the low-energy behavior of this vortex system, we consider fields which are linear combinations of these two low-energy modes,

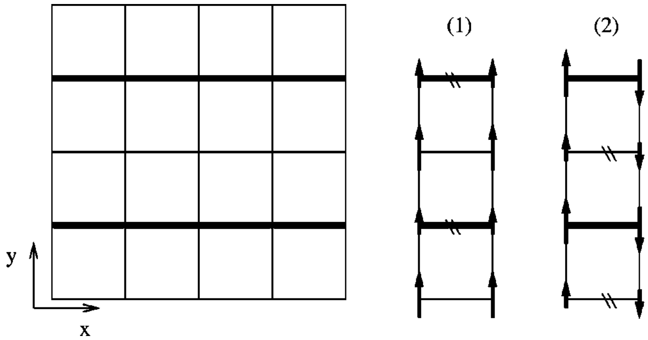

FIG. 2. Representation of the fully frustrated 2D lattice. Dark lines show the location of negative or ''antiferromagnetic'' bonds ($t_{r,r'}<0$). The two low-energy modes χ_r^0 and χ_r^π are shown in (1) and (2), respectively. Long arrows have length $2+\sqrt{2}$ and short arrows have length $\sqrt{2}$. Frustrated or ''unhappy'' bonds are marked with a slash.

$$\Phi(\vec{r},\tau)=\Psi_0(\vec{r},\tau)\chi_{\vec{r}}^0+\Psi_\pi(\vec{r},\tau)\chi_{\vec{r}}^\pi. \quad (17)$$

We now have two complex fields $\Psi_0(\vec{r})$ and $\Psi_\pi(\vec{r})$, which describe the low-energy configurations of our vortex system. The phase transitions of the system can be explored within Ginzburg-Landau theory. The Ginzburg-Landau Hamiltonian for the two-vortex system must preserve all the symmetries of the original lattice Hamiltonian, namely, discrete $\hat{x}$ and $\hat{y}$ translations, rotations by $\pi/2$, and the vortex U(1) symmetry ($\Phi\to e^{i\alpha}\Phi$), as well as Hermiticity. In terms of our two complex vortex fields, these symmetry transformations take a simpler form when expressed in terms of the fields

$$\phi_1=\Psi_0+i\Psi_\pi, \quad \phi_2=\Psi_0-i\Psi_\pi \quad (18)$$

as follows:

$$T_{\hat{x}}:\phi_1\leftrightarrow\phi_2; \quad (19)$$

$$T_{\hat{y}}:\phi_1\to i\phi_2, \quad \phi_2\to -i\phi_1; \quad (20)$$

$$R_{\pi/2}:\phi_1\to e^{i\pi/4}\phi_1, \quad \phi_2\to e^{-i\pi/4}\phi_2; \quad (21)$$

$$\mathrm{U}(1):\phi_a\to e^{i\alpha}\phi_a \ \ (\text{for } a=1 \text{ and } 2). \quad (22)$$

Allowed terms for the action include

$$(\mathrm{I}):\ (|\phi_1|^2)^n+(|\phi_2|^2)^n,$$

$$(\mathrm{II}):\ (|\phi_1|^2|\phi_2|^2)^n,$$

$$(\mathrm{III}):\ [(\phi_1^*\phi_2)^4+(\phi_1\phi_2^*)^4]^n$$

(with arbitrary positive integer n), and combinations of these terms. Expanding in powers of the fields, we take as our Landau-Ginzburg action

$$S_{LG}=\int d^2x\,d\tau\sum_{a=1,2}[|\partial_\mu\phi_a|^2+r|\phi_a|^2]+u_4\left(\sum_a |\phi_a|^2\right)^2 + v_4|\phi_1|^2|\phi_2|^2-v_8[(\phi_1^*\phi_2)^4+\mathrm{H.c.}], \quad (23)$$

where τ has been rescaled to set the vortex velocity $v_v=1$. The terms labeled by u_4 and v_4 are the only allowed quartic terms, and are invariant under independent U(1) transformations on ϕ_1 and ϕ_2. We have kept the v_8 term because it is the lowest-order term which breaks this symmetry down to the global U(1) of Eq. (22). This model will be employed to construct a description of various phases proximate to the d-wave superconductor within mean-field theory.

We wish to characterize the various states of this vortex system. It is important to emphasize at this point that not all vortex condensates destroy superconductivity. Superconductivity is destroyed when the dual U(1) symmetry of the vortex theory [Eq. (22)] is broken. Therefore, it is possible to have nontrivial vortex condensates which are superconducting. This leads to two scenarios for the superconductor-insulator transition at half-filling. First, we may consider superconductors which are described by a vacuum of vortices; superconductivity is then destroyed when single vortices proliferate and condense [in a way which breaks the dual U(1)]. Alternatively, the superconducting state could itself be a U(1)-preserving vortex condensate which then undergoes a transition which breaks the dual U(1), destroying superconductivity.

In the following sections, we explore the phases of our dual-vortex model using the Landau-Ginzburg action of Eq. (23). Due to the frustration of the vortex theory with one electron per unit cell, the vortex condensates will break lattice symmetries. Some of these spatially ordered states are superconductors and some are insulators. We will begin by describing the possible superconducting states within the dual theory (including a *striped* superconductor), and then move on to a description of the insulating states. Ignoring charge fluctuations in the superconducting states [as we have in arriving at Eq. (23)] is not justified, and a good description of these states would require putting the charge fluctuations back in. Here, we content ourselves to characterizing the phases of our vortex system by their broken symmetries. We conclude with a summary of the possible transitions from superconductor to insulator within this mean-field theory.

A. Superconductors

1. Vortex vacuums

The simplest superconducting phase is just the vortex vacuum. This is the standard BCS d-wave superconductor. Destruction of superconductivity occurs when single vortices proliferate out of the vacuum and condense, breaking the dual U(1) symmetry. The effective action for this transition is Eq. (23).

2. Paired vortex condensates

Condensation of single $hc/2e$ vortices necessarily breaks the dual U(1) symmetry [Eq. (22)] destroying superconductivity. However, when *pairs* of vortices condense, the U(1) can be preserved. Consider the paired vortex condensate

$$\langle \phi_2^* \phi_1 \rangle \neq 0,\ \langle \phi_1 \rangle = \langle \phi_2 \rangle = 0. \tag{24}$$

We see that in this condensate the dual U(1) is preserved, and the state is characterized by the phase of the condensate (setting the amplitude $|\langle \phi_2^* \phi_1 \rangle| = 1$ for simplicity),

$$\langle \phi_2^* \phi_1 \rangle = e^{i\theta}, \tag{25}$$

$$\theta \equiv \theta_1(x) - \theta_2(x). \tag{26}$$

Here, θ_1 and θ_2 are the phases of ϕ_1 and ϕ_2, respectively, and are still free to fluctuate. Only the combination $\theta = \theta_1 - \theta_2$ is uniform, reflecting the fact that the dual U(1) symmetry is preserved (i.e., ϕ_1 and ϕ_2 are uncondensed). The only term in the Landau-Ginzburg action which depends on θ is the v_8 term, giving

$$S_v = -v_8 \int d^2x\, d\tau \cos(4\theta). \tag{27}$$

We see that the ground state depends on the sign of v_8:

$$v_8 > 0 : \theta = n\frac{\pi}{2}, \tag{28}$$

$$v_8 < 0 : \theta = \frac{\pi}{4} + n\frac{\pi}{2}, \tag{29}$$

with n an integer.

The spatial symmetries in Eqs. (19)–(21), written in terms of the relative phase θ, are

$$T_{\hat{x}} : \theta \to -\theta, \tag{30}$$

$$T_{\hat{y}} : \theta \to \pi - \theta, \tag{31}$$

$$R_{\pi/2} : \theta \to \theta + \frac{\pi}{2}. \tag{32}$$

From this we can see that the vortex condensate favored by $v_8 > 0$ breaks the lattice rotational symmetry and *one* of the two translational symmetries. We therefore associate this condensate with a stripe-type ordering: a *striped superconductor*. This state is particularly interesting given recent experimental results which suggest possible stripes in the superconducting state of $La_{2-x}Sr_xCuO_4$.[11] The ground state for $v_8 < 0$ breaks all of the lattice symmetries; we identify this state with a ''plaquette'' order which will be made more explicit in upcoming sections when we discuss the insulating states of the vortex system. For now, we emphasize the possibility of spatially ordered superconducting states which emerge quite naturally within our dual vortex description.

Still working in the dual description, these striped and plaquette superconductors are described by an effective theory of *one* vortex species, since the paired condensation has locked the two original vortices together: the vortex phases $\theta_1(x)$ and $\theta_2(x) = \theta_1(x) - \theta$ still fluctuate within the superconducting phase, but not independently. When the remaining phase θ_1 becomes constant over the sample, the dual U(1) is broken, and superconductivity is destroyed. Therefore, for these spatially ordered superconductors, S_{LG} [Eq. (23)] reduces to

$$S_v = \int d^2x\, d\tau [|\partial_\mu \phi_1|^2 + r|\phi_1|^2 + u(|\phi_1|^2)^2]. \tag{33}$$

It is worth noting that we have gone from a theory of a single fully frustrated vortex to a theory of a single unfrustrated vortex via a theory of two vortices. This is possible because in a striped or plaquette superconductor, the unit cell is doubled. If one started from scratch in constructing a dual theory of these striped (plaquette) superconductors, the vortices would see 2π rather than π flux per (doubled) unit cell and there would be only one low-energy mode.

B. Confined insulators

When single vortices condense at half-filling, we move from the *d*-wave superconductor into a confined insulator. Within our dual formulation, these insulators are described by condensates which break the dual U(1) symmetry of Eq. (22). In the case of superconductors which are vortex vacuums, because we have *two* vortex species, there are many ways to do this and therefore many possible single-vortex condensates. We will see that these different vortex condensates correspond to different insulating states of electrons. We return to the case of the striped and plaquette superconductors after first enumerating the insulating states at the mean-field level, using the action of Eq. (23).

The most general U(1)-breaking vortex condensate is

$$\langle \phi_1 \rangle = |\langle \phi_1 \rangle| e^{i\theta_1}, \tag{34}$$

$$\langle \phi_2 \rangle = |\langle \phi_2 \rangle| e^{i\theta_2}, \tag{35}$$

where $|\langle \phi_1 \rangle|$, $|\langle \phi_2 \rangle|$, θ_1, and θ_2 are all fixed real numbers. Within our dual Landau-Ginzburg model, condensing the vortices corresponds to setting $r < 0$ and $u_4 > 0$. The signs of v_4 and v_8 then determine the ground state. For $v_4 < 0$, both vortex species acquire a nonzero amplitude $|\langle \phi_1 \rangle| = |\langle \phi_2 \rangle| \neq 0$ and their relative phase $\theta_{12} = \theta_1 - \theta_2$ is determined by the sign of v_8. On the other hand, if $v_4 > 0$, the ground states are condensates of either ϕ_1 *or* ϕ_2 and the sign of v_8 is irrelevant. Each of these condensates will correspond to a different insulating state of the electron system. We consider each case in turn.

1. $|\langle \phi_1 \rangle| = |\langle \phi_2 \rangle| \neq 0$

These condensates are favored by $v_4 < 0$, and the relative phase $(\theta_1 - \theta_2)$ is determined by the sign of v_8. Taking the magnitudes $|\langle \phi_1 \rangle| = |\langle \phi_2 \rangle| = 1$, this term in the action can be rewritten as

$$-v_8[(\phi_1^* \phi_2)^4 + \text{H.c.}] = -v_8 \cos(4\theta_{12}). \tag{36}$$

In terms of this relative phase, the spatial symmetries are given by Eqs. (30)–(32) with the replacement $\theta \to \theta_{12}$.

a. $\theta_1 - \theta_2 = n\pi/2$. This class of condensates is preferred by $v_8 > 0$. There are four general states, corresponding to

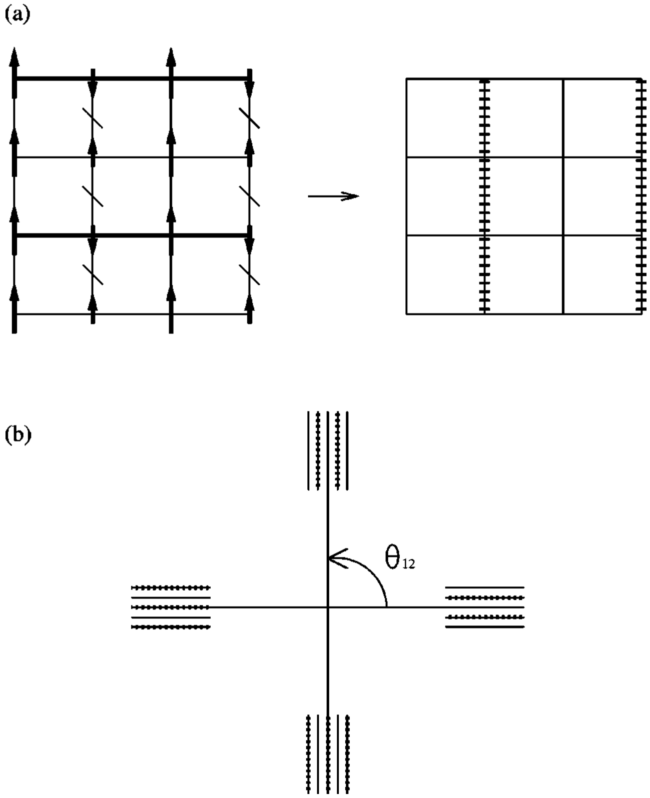

FIG. 3. (a) The vortex condensates with $\theta_{12}=\pi/2$, which correspond to $\Phi(r)=(1/\sqrt{2})(\chi_r^0+\chi_r^\pi)$. Frustrated bonds are slashed. (b) Schematic of the four ''striped'' states corresponding to $\theta_{12}=n\pi/2$ with higher-energy (frustrated) bonds slashed.

each of the possible values of n. We see by the symmetry transformations in Eqs. (30)–(32) that each of these states breaks the lattice rotational symmetry as well as breaking *one* of the two translational symmetries while leaving the other intact. On these grounds alone, we could guess that these states correspond to ''stripelike'' phases. To be more concrete, we may go back to our real-space representation for the vortex field $\Phi(\vec{r})$ in Eq. (17) and draw real-space pictures of these lattice states. The values of the fields at various points will be gauge dependent, but the location of frustrated bonds (which are places of higher energy density) is gauge independent and therefore a good way to characterize the state of the system. This is shown for the case $\theta_{12}=\pi/2$, as an example, in Fig. 3(a). Investigations of this sort lead us to conclude that the four ground states of the system in this case are characterized by ''stripes'' of energy density as shown in Fig. 3(b).

We now turn to a characterization of this system in terms of the electron degrees of freedom. Because we have broken the dual U(1) symmetry of the vortices and we are at half-filling, these states will be Mott insulators. The charge degrees of freedom reside on the plaquettes of the dual lattice and are fixed at one charge of e per dual plaquette. It has been suggested[12,4] that the frustrated bonds of the dual lattice should correspond to singlet bonds of the electron system, since one expects regions of higher energy along the links where the electrons spend most of their time. This relationship between the frustrated bonds on the dual lattice and the singlet bonds on the original lattice is illustrated in Fig. 4. These ''striped'' vortex phases then correspond to spin-Peierls (or ''bond-density-wave'') order in the insulating electron system.

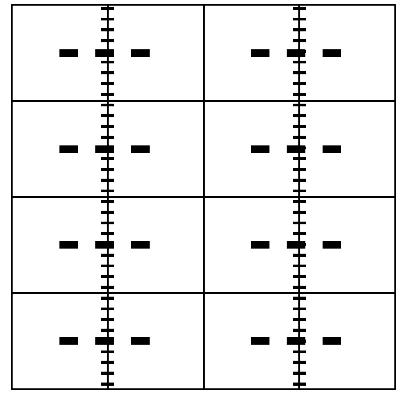

FIG. 4. Schematic of the relationship between frustrated bonds of the dual lattice (slashed) and links of the original lattice where the corresponding ''singlet bonds'' reside (dashed).

b. $\theta_1-\theta_2=\pi/4+n\pi/2$. These condensates are favored by $v_8<0$. Here, however, each ground state breaks all of the discrete lattice symmetries. We proceed as above and obtain characterizations of these states in terms of the location of frustrated bonds. The result is a plaquettelike structure, as seen in Fig. 5.

In terms of the electron degrees of freedom, we would like to again interpret the frustrated bonds of the dual lattice as regions where singlet-type bonds of the electron system reside. The plaquettelike structure of these vortex states may then correspond to a ''plaquette RVB'' state of the electron system, as shown in Fig. 6.

2. $\langle\phi_1\rangle\neq 0, \langle\phi_2\rangle=0$ or $\langle\phi_1\rangle=0, \langle\phi_2\rangle\neq 0$

These condensates are preferred in the case $v_4>0$. We may proceed as above in drawing real-space diagrams corresponding to these states. We find, as shown in Fig. 7, that these states have vortex currents around each plaquette, of alternating sign.

In order to interpret this state, we will have to put back in the spinons which have been ignored in the previous discussion. The vortex-spinon action is

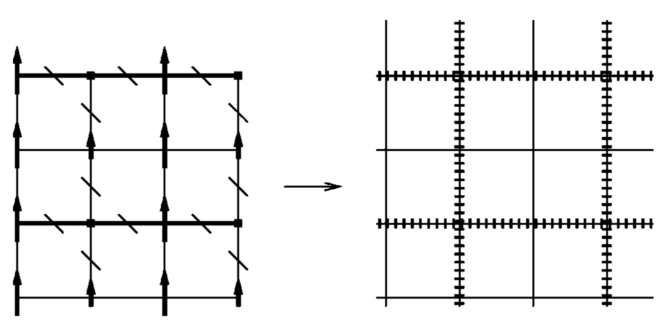

FIG. 5. The vortex condensate with $\theta_{12}=\pi/4$, which corresponds to $\Phi_r=0.9\chi_r^0+0.4\chi_r^\pi$. Locations where the field is zero are denoted by a dot. Relatively unhappy bonds are slashed.

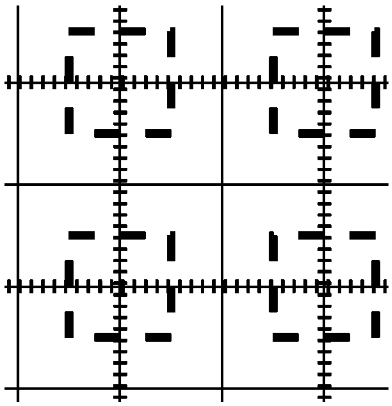

FIG. 6. Schematic of states corresponding to the $v_8<0$ vortex condensates. Relatively unhappy bonds of the dual lattice are slashed, and links of the original lattice where the singlet-type bonds reside are dashed. We associate this structure with a ''plaquette RVB'' state of the electrons.

$$S=S_v+S_s+S_{CS}, \tag{37}$$

$$S_v=-t_v\sum_{\langle i'j'\rangle}\mu_{i'j'}\cos\left(\theta_{i'}-\theta_{j'}-\frac{a_{i'j'}}{2}\right), \tag{38}$$

$$\text{with } \langle\nabla\times\mathbf{a}\rangle=2\pi, \tag{39}$$

$$S_s=-\sum_{\langle ij\rangle}\sigma_{ij}[t^s_{ij}\overline{f_i}f_j+t^\Delta_{ij}f_{i\uparrow}f_{j\downarrow}]-\sum_i\overline{f_i}f_i, \tag{40}$$

$$S_{CS}=\sum i\frac{\pi}{4}\left(1-\prod_{\square'}\mu\right)(1-\sigma_{ij}), \tag{41}$$

where i,j label sites on the original lattice and i',j' label sites on the dual lattice. Looking at S_v, we see that the alternating vortex currents would like to induce compensating fluctuations in either the $a_{i'j'}$ or $\mu_{i'j'}$ fields. Allowing fluctuations of the gauge field $\mathbf{a}$ (which describes charge fluctuations), and ignoring the coupling to the spinons, the alternating vortex currents would induce charge-density-wave (CDW) order at wave vector (π,π). However, with a large on-site U, this state will be greatly suppressed. If we forbid charge fluctuations, we see that the alternating vortex currents will instead drive a mean field in the Z_2 gauge field:

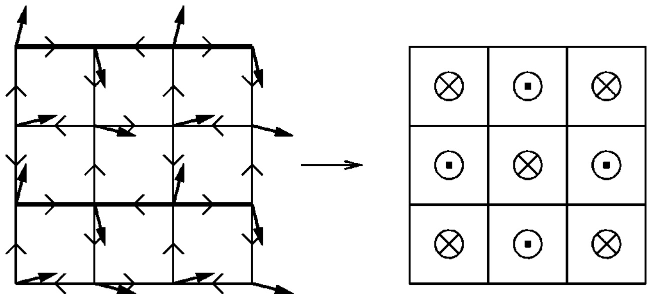

FIG. 7. The vortex condensate with $\Phi_r=\chi^0_r+i\chi^\pi_r$. Since the field is complex, the magnitude is given by the length and the argument by the direction of the arrow at each site. The direction of vortex current is indicated on the bonds.

$$\prod_{\square'}\mu_{i'j'}=(-1)^{n_f}\approx-1 \tag{42}$$

(where n_f is the number of spinons in the dual plaquette denoted by $\square$), which corresponds to one spinon per unit cell. Unlike the previously considered vortex condensates (with $v_4<0$), at the level of vortex mean-field theory, this state has no broken translational symmetries. (However, we cannot rule out the breaking of symmetries by the charge and spin fluctuations.) We note that a possible candidate for this state which has one electron per until cell and uniform energy density is the antiferromagnet.

C. Summary of vortex theory

We have seen that our dual-vortex theory describes both standard BCS and striped or plaquette *d*-wave superconduct-

TABLE I. Summary of vortex condensates, listing the spatial symmetries (translations in the $\hat{x}$ or $\hat{y}$ directions and rotation by 90°) broken by each. The first two condensates preserve the dual U(1) symmetry and are therefore superconductors; the remaining three break the dual U(1) and are confined insulators.

Vortex condensate		Broken spatial symmetries	Characterization of phase
$\langle\phi_2^*\phi_1\rangle=e^{i\theta}$	$\theta=n\frac{\pi}{2}$	$R_{\pi/2}$, T_x, or T_y	Striped SC
	$\theta=\frac{\pi}{4}+n\frac{\pi}{2}$	$R_{\pi/2}$	Plaquette SC
$\lvert\langle\phi_1\rangle\rvert=\lvert\langle\phi_2\rangle\rvert$	$\theta_1-\theta_2=n\frac{\pi}{2}$	$R_{\pi/2}$, T_x or T_y	Striped confined insulator
	$\theta_1-\theta_2=\frac{\pi}{4}+n\frac{\pi}{2}$	$R_{\pi/2}$	Plaquette confined insulator
$\langle\phi_1\rangle\neq0$, $\langle\phi_2\rangle=0$ or $\langle\phi_1\rangle=0$, $\langle\phi_2\rangle\neq0$		None	Confined insulator

ors as well as a host of confined insulating states. A summary of the various vortex condensates is given in Table I. We now summarize the results of Landau theory for the transitions from the d-wave superconductor to the confined insulator at half-filling.

We consider first the transition from a vacuum of vortices (a superconductor) to a U(1)-breaking condensate of vortices (an insulator). Within mean-field theory, the nature of the insulating state is determined by the signs of the coupling constants in Eq. (23), and we have the following possible *direct* transitions out of the symmetric d-wave superconductor:

$$v_4<0,\ v_8>0:\ \ \text{dSC}\rightarrow\text{spin}-\text{Peierls}, \tag{43}$$

$$v_4<0,\ v_8<0:\ \ \text{dSC}\rightarrow\text{plaquette RVB}, \tag{44}$$

$$v_4>0:\ \ \text{dSC}\rightarrow\text{uniform state of electrons}. \tag{45}$$

One might hope to ascertain which of these insulating states is preferred close to a d-wave superconductor, including fluctuations beyond the mean-field level, by considering the fixed points of the action in Eq. (23). In particular, we see that the sign of v_4 determines whether we enter one of the states of broken translational symmetry (spin-Peierls or plaquette RVB) or the state with uniform energy density (possibly the antiferromagnet). The work of Blagoeva[13] on the theory of two-component complex fields with these (and other) couplings gives a stable fixed point at $v_4<0$, to order ϵ^2 ($\epsilon=4-D$, $D=d+1$, in d spatial dimensions). This suggests that the transition dSC $\rightarrow$ spin-Peierls would be preferred over dSC $\rightarrow$ uniform state. This is tantalizing given the experimental evidence for intervening ‘‘stripey’’ phases between the superconducting and antiferromagnetic phases in the cuprates.[14]

In the case of the striped and plaquette superconductors, when the single-vortex species in Eq. (33) condenses, superconductivity in these states is destroyed and we enter a confined insulating state. Because the relative phase θ_1-θ_2 is already fixed within these superconductors, we see from our above analysis of the insulating phases that the insulating state is pre-determined. The striped superconductor (with $\theta=n\pi/2$) enters the spin-Peierls insulator, and the plaquette superconductor (with $\theta=\pi/4+n\pi/2$) enters the plaquette-RVB insulator. In other words, these spatially ordered superconductors make transitions into insulating states with the same broken spatial symmetries:

$$v_8>0:\ \ \text{striped SC}\rightarrow\text{spin-Peierls}, \tag{46}$$

$$v_8<0:\ \ \text{plaquette SC}\rightarrow\text{plaquette-RVB}. \tag{47}$$

In the preceding section, we have considered states of electron systems at half-filling near a d-wave superconductor within a dual formulation in terms of vortices. Each phase is characterized by a dual (vortex) order parameter. At one electron per unit cell, the frustration of the vortex theory manifests itself in spontaneously broken spatial symmetries. Exploiting the fact that the vortex order parameters break spatial symmetries has helped us identify these vortex phases with more familiar phases of electrons (such as the spin-Peierls state), as well as phases like the striped superconductor. The dual formulation shows us the enhanced chance for striped superconductors near half-filling. In the next section, we will add back in the spinons (and, along with them, their long-ranged statistical interaction with the vortices), and extract information about the critical properties of the confinement transition using field theory methods.

IV. CONFINEMENT TRANSITION

Having explored the vortex sector of the theory with one electron per unit cell, we now wish to put the spinons back in and address the critical properties of the confinement transition. Because we will continue to work at half-filling, the confined states of electrons will be Mott insulators. While the theory of vortices and spinons coupled to Z_2 gauge fields may in principle be numerically accessible, the action suffers from the notorious fermion sign problem. Here, we discuss a special case which will turn out to be accessible to perturbative RG calculations.

Focusing on the spinon Hamiltonian (and dropping the Z_2 gauge field for the time being)

$$H_s=-\sum_{\langle rr'\rangle}[t^s_{rr'}f^\dagger_r f_{r'}+t^\Delta_{rr'}(f_{r\uparrow}f_{r'\downarrow}+\text{H.c.})], \tag{48}$$

we choose the special case

$$t^s=|t^\Delta|\equiv t, \tag{49}$$

with nearest-neighbor d-wave pairing amplitude

$$t^\Delta_{\vec r,\vec r\pm\hat x}=+t,$$
$$t^\Delta_{\vec r,\vec r\pm\hat y}=-t. \tag{50}$$

Following Affleck *et al.*,[15] we introduce the fields

$$\begin{pmatrix} d_{r\uparrow}\\ d^\dagger_{r\downarrow}\end{pmatrix}=\begin{cases} e^{-i(\pi/8)\sigma_y}\begin{pmatrix} f_{r\uparrow}\\ f^\dagger_{r\downarrow}\end{pmatrix}, & \text{for } y \text{ even},\\[2ex] (-i\sigma_y)e^{-i(\pi/8)\sigma_y}\begin{pmatrix} f_{r\uparrow}\\ f^\dagger_{r\downarrow}\end{pmatrix}, & \text{for } y \text{ odd}\end{cases} \tag{51}$$

(where σ_y is the usual Pauli matrix); the spinon Hamiltonian becomes

$$H_s=-\sum_{\langle rr'\rangle}t_{rr'}(d^\dagger_{r\alpha}d_{r'\alpha}+\text{H.c.}), \tag{52}$$

with

$$t_{rr'}=\begin{cases}-t & \text{for } y \text{ and } y' \text{ even},\\ t & \text{otherwise.}\end{cases} \tag{53}$$

This is the Hamiltonian of fermions hopping in 2D in the presence of π flux per plaquette. We have succeeded in finding a Hamiltonian for the spin sector which has a conserved fermion number. The original theory [Eq. (4)] can now be written in terms of these d fermion fields, the chargons, and the Z_2 gauge field. Following a transformation which can get rid of the Berry's phase term[16] this theory can be modeled numerically with no fermion sign problem. Here, we instead proceed to a low-energy continuum Hamiltonian for the spin sector. To this end, we diagonalize H_s to find two Dirac points. These are the usual d-wave quasiparticle nodes at $(k_x,k_y)=(\pm\pi/2,\pm\pi/2)$ except that, due to the π flux per plaquette, we have doubled the unit cell and halved the Brillouin zone; it now contains only two of these nodes, which we denote $\vec{K}_1$ and $\vec{K}_2$. In terms of long-wavelength fields residing at these two nodes,

$$d_{j\alpha}(\vec{x})\simeq\psi_{1j\alpha}(\vec{x})e^{iK_1\cdot x}+\psi_{2j\alpha}(\vec{x})e^{iK_2\cdot x} \tag{54}$$

(where $j=1,2$ labels the sublattice), the continuum Hamiltonian is

$$H_s=\int d^2x\, v_s\psi^\dagger_{1\alpha}[\tau_1(-i\partial_x)+\tau_2(-i\partial_y)]\psi_{1\alpha} + v_s\psi^\dagger_{2\alpha}[\tau_2(-i\partial_x)+\tau_1(-i\partial_y)]\psi_{2\alpha}, \tag{55}$$

where

$$\tau_1\equiv\frac{1}{\sqrt{2}}(\tau_x+\tau_z), \tag{56}$$

$$\tau_2\equiv\frac{1}{\sqrt{2}}(\tau_x-\tau_z). \tag{57}$$

Here, $\vec{\tau}$ acts in the sublattice space, and we have rotated the x and y axes at each node by 45°. Just as the Hamiltonian for the d fermions was diagonal in the spin label, so is this one, and we are left with a theory of four species of Dirac fermions. Note that the spinon characteristic velocity v_s is isotropic in space because of our choice $t_s=|t_\Delta|$.

Defining Dirac matrices in 2+1 dimensions,

$$\begin{aligned}&\text{at node } \vec{K}_1: \quad \text{at node } \vec{K}_2:\\ &\gamma_0\equiv\tau_y, \quad \gamma_0\equiv\tau_y,\\ &\gamma_1\equiv\tau_2, \quad \gamma_1\equiv-\tau_1,\\ &\gamma_2\equiv-\tau_1, \quad \gamma_2\equiv\tau_2,\end{aligned} \tag{58}$$

$$(\gamma_\mu)^\dagger=\gamma_\mu;\ \ \{\gamma_\mu,\gamma_\nu\}=2\delta_{\mu\nu}\ \ (\text{at each node}), \tag{59}$$

we proceed to the Euclidean Lagrangian density

$$\mathcal{L}_s=\overline{\psi_a}[\gamma_0\partial_0+v_s\gamma_i\partial_i]\psi_a, \tag{60}$$

$$\overline{\psi}\equiv\psi^\dagger\gamma_0. \tag{61}$$

The fields ψ_a have two components (corresponding to the sublattice label), and summation conventions on the number of species $a\in[1,4]$ (one for each spin at each of the two nodes) and the spatial dimension $i\in[1,2]$ are in use.

We have succeeded in writing a low-energy effective theory for the spin sector which is just that of four species of two-component Dirac fermions. We may now write down a full low-energy effective theory where, since the spinon and vortex sectors each display U(1) symmetries

$$\psi_a\rightarrow e^{i\alpha_\psi}\psi_a, \tag{62}$$

$$\phi\rightarrow e^{i\alpha_\phi}\phi, \tag{63}$$

we may implement the statistical spinon-vortex interaction using U(1) (rather than Z_2) Chern-Simons gauge fields $\mathbf{A}^\phi$ and $\mathbf{A}^\psi$. We proceed to a field theory modeling the confinement transition between a spin-charge-separated (d-wave) superconductor and a spin-charge-confined Mott insulator. For simplicity, we consider the vortex theory with only one species [Eq. (33)], and thereby consider transitions out of the striped-plaquette superconductor given in Eqs. (46) and (47). The low-energy effective action is

$$S=\int d^2x\,d\tau[\mathcal{L}_s+\mathcal{L}_v+\mathcal{L}_{CS}+\mathcal{L}_{vs}], \tag{64}$$

$$\mathcal{L}_s=\overline{\psi_a}(\not{\partial}-ig\not{A}^\psi)\psi_a+\kappa\overline{\psi_a}(\gamma_i\partial_i-ig\gamma_iA^\psi_i)\psi_a, \tag{65}$$

$$\mathcal{L}_v=|(\partial_\mu-igA^\phi_\mu)\phi|^2+m^2|\phi|^2+u_0(|\phi|^2)^2, \tag{66}$$

$$\mathcal{L}_{CS}=i\epsilon_{\mu\nu\lambda}A^\phi_\mu\partial_\nu A^\psi_\lambda, \tag{67}$$

$$\mathcal{L}_{vs}=v_0\psi^\dagger_a\psi_a|\phi|^2, \tag{68}$$

with

$$a\in[1,N=4],\quad \kappa=\frac{v_s}{v_v}-1, \tag{69}$$

where κ is a measure of the velocity anisotropy between vortices and spinons and will be treated as a perturbation. We have added the term $\mathcal{L}_{vs}$ in the interest of including all possible relevant interactions. The Chern-Simons term causes a vortex taken around a spinon to acquire a phase of

$$\phi\rightarrow\exp\left(ig\oint\vec{A}^\phi\cdot d\vec{l}\right)\phi=e^{ig^2}\phi, \tag{70}$$

and likewise for a spinon after encircling a vortex,

$$\psi\rightarrow\exp\left(ig\oint\vec{A}^\psi\cdot d\vec{l}\right)\psi=e^{ig^2}\psi, \tag{71}$$

so that the full statistical interaction is achieved when

$$g^2=\pi=2\pi\alpha,\quad \alpha=\frac{1}{2} \tag{72}$$

(where α is the so-called ''statistics angle'' and is equal to 1/2 since the vortex and the spinon are relative semions). The theory as written neglects charge fluctuations, which is not justified within the superconducting phase. The full vortex theory would include an additional minimal coupling to a gauge field $\mathbf{a}$.[17] As seen in the dual XY model, this coupling causes runaway flows, and is probably best modeled numerically. At this point, we leave out the gauge field $\mathbf{a}$ and its attendant problems, but we will revisit this question shortly.

When the vortex Lagrangian is taken through criticality ($m^2<0$), the statistical interaction, mediated by the gauge fields A_μ^ϕ and A_μ^ψ, will drive spin-charge confinement. Here, we seek the effect of these statistics on critical properties of the system. In particular, we wish to calculate β functions for the couplings u_0, v_0, κ, and g, as well as the anomalous dimensions of the vortex and spinon fields.

We work in $D=d+1=3$ dimensions (indeed, our Chern-Simons flux attachment is not well defined in higher dimensions), and define dimensionless couplings

$$u=\Lambda^{-1}u_0K_{D=3}, \tag{73}$$

$$v=v_0K_{D=3}, \tag{74}$$

where factors of $K_D=[2^{D-1}\pi^{D/2}\Gamma(D/2)]^{-1}$ have been put in for later convenience. The bare propagators in the Landau gauge are

$$\text{fermions:}\quad G_0^\psi=-\frac{i\not{k}}{k^2}, \tag{75}$$

$$\text{vortices:}\quad G_0^\phi=\frac{1}{k^2}, \tag{76}$$

$$\text{gauge fields:}\quad S_0^{\mu\nu}=-\frac{\epsilon^{\mu\nu\lambda}k^\lambda}{k^2}=\langle A_\mu^\psi A_\nu^\phi\rangle, \tag{77}$$

$$\langle A^\phi A^\phi\rangle=\langle A^\psi A^\psi\rangle=0. \tag{78}$$

(The fermion propagator is diagonal in the label a, so we have suppressed this index.)

For the β functions we find, to lowest nonvanishing order (one loop),

$$\frac{du}{dl}=u-10u^2+\left(\frac{N}{3}+C\kappa\right)v^2+\cdots, \tag{79}$$

$$\frac{dv}{dl}=-4uv+\cdots, \tag{80}$$

$$\frac{dg^2}{dl}=0, \tag{81}$$

$$\frac{d\kappa}{dl}=0+\cdots. \tag{82}$$

We expect that at higher orders, g will enter into du/dl and dv/dl nontrivially, but that g itself should not renormalize at any order, following the argument given by Semenoff *et al.*[18] The one-loop RG equations for u and v have a stable solution at $v=0$, so that the theory decouples into separate spinon and vortex theories. At this order, since the spinon and vortex sectors decouple, we may ignore the Chern-Simons gauge fields (effectively taking $g=0$) and include the effects of charge fluctuations by using the full dual XY model for the vortex sector:

$$\mathcal{L}_v=|(\partial_\mu-ie_0a^\mu)\phi|^2+\frac{1}{2}|\vec{\nabla}\times\vec{a}|^2+m^2|\phi|^2+u_0(|\phi|^2)^2. \tag{83}$$

Recently, much work has gone into tackling the critical properties of the $e\neq 0$ model,[19] and we may use these results.

To first order, then, we find a *fixed line*, parametrized by values of the statistics angle α (or, equivalently, the coupling g). At lowest nonvanishing order, this line is given by

$$u^*\simeq u^*_{dual}, \tag{84}$$

$$v^*\simeq 0, \tag{85}$$

$$(g^2)^*=g^2=\pi, \tag{86}$$

$$e^*\simeq e_{dual}, \tag{87}$$

$$\kappa^*\simeq\kappa, \tag{88}$$

where by u^*_{dual} and e_{dual} we mean the values of these couplings at the fixed point of the dual XY model.

In order to see whether spinon-vortex velocity anisotropy grows, we need to take the β function for κ to its lowest nonvanishing order, which is two loops. The result is

$$\frac{d\kappa}{dl}=-\frac{31}{240}\frac{g^4}{\pi^2}\kappa+\cdots. \tag{89}$$

Since the system flows toward $\kappa=0$, it is legitimate to treat this term as a perturbation, and the theory becomes ''relativistic'' at the critical point.

We proceed by calculating the anomalous dimensions of the spinon and vortex fields, to lowest order, near the critical point. To that end, we consider the self-energies

$$[G^\phi(k)]^{-1}=[G_0^\phi(k)]^{-1}+\Sigma^\phi(k), \tag{90}$$

$$[G^\psi(k)]^{-1}=[G_0^\psi(k)]^{-1}+\Sigma^\psi(k). \tag{91}$$

Near the critical point, the anomalous dimensions are given by

$$G^\phi(k)\propto\frac{1}{|k|^{2-\eta_\phi}}, \tag{92}$$

$$G^\psi(k)\propto\frac{-i\not{k}}{|k|^{2-\eta_\psi}} \tag{93}$$

(up to additive constants). Working at the fixed point $(u,v,g^2)=(u^*=u^*_{dual},\ v^*=0,\ g^{2*}=\pi)$ and calculating the spinon and vortex self-energies to two loops in three dimensions, we find

$$\eta_\phi=\eta_{dual}-\frac{4}{3}\frac{(g^4)^*}{16\pi^2}N+\cdots, \tag{94}$$

$$\eta_\psi=-\frac{1}{3}\frac{(g^4)^*}{16\pi^2}+\cdots. \tag{95}$$

Since we are in the case with one vortex species, we may take the numerical results of Hove and Sudbø[19] for the anomalous dimension of the vortex field in the *full* dual XY model in $D=3$: $\eta_\phi \simeq -0.24$. After plugging in $N=4$ and $g^{2*}=\pi$ into our result, we find

$$\eta_\phi \simeq -0.24 - \frac{1}{3} \simeq -0.57, \tag{96}$$

$$\eta_\psi \simeq -\frac{1}{48} \simeq -0.02. \tag{97}$$

These critical exponents may reveal themselves in many quantities. In particular, the spectral function as probed by ARPES and the spin-spin correlations probed by NMR or neutron scattering. Within our theory, the low-energy electron correlator decouples into chargon and spinon pieces for $g \to 0$:

$$\langle c(x) c^\dagger(0) \rangle = \langle b(x) b^\dagger(0) \rangle \langle f(x) f^\dagger(0) \rangle. \tag{98}$$

These correlators will exhibit anomalous dimensions η_b and η_f, which can be expanded perturbatively around $g^*=0$:

$$\eta_b = \eta_{XY} + C_b^{(2)} (g^*)^2 + \cdots, \tag{99}$$

$$\eta_f = \eta_\psi = C_f^{(2)} (g^*)^2 + \cdots, \tag{100}$$

where we have calculated $C_f^{(2)} \simeq -0.03$. The anomalous dimension for the 3D XY model (appropriate for one vortex species) has been calculated by Hasenbusch and Török using Monte Carlo methods;[20] they find $\eta_{XY} \simeq 0.038$. The anomalous dimension will also enter into the spin-spin correlation function. Within our model, it looks as though vertex correction diagrams will not contribute as much near the critical point as the direct $[G_\psi]^2$ term.

V. CONCLUSIONS

In this paper, we have used a gauge theory of strongly interacting electrons to explore the regions near the superconducting state in the high-T_c cuprates. This gauge theory exhibits spin-charge-separated and spin-charge-confined phases. We have seen that the presence of one electron per unit cell has profound implications for the regions near the superconducting state. Within a dual description, half-filling of electrons corresponds to fully frustrated vortices, leading to a spontaneous breaking of translational symmetries in the electron system. From this, we have seen the possibility of striped superconductivity as well as a host of confined insulators descending from *d*-wave superconducting phases. We have then used Chern-Simons methods to calculate lowest-order critical properties of the confinement transition between these phases. Because we have worked at half-filling of electrons throughout, our results are of particular relevance to the undoped cuprate materials, which may be spin-charge confined. However, we also hope that the flavor of our results may be of interest in the heavily overdoped materials, where the confinement of spin and charge may result in a Fermi liquid phase.

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