

## Detecting fractions of electrons in the high- $T_c$ cuprates

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(Received 18 December 2000; published 12 November 2001)

We propose several tests of the idea that the electron is fractionalized in the underdoped and undoped cuprates. These include the ac Josephson effect and tunneling into small superconducting grains in the Coulomb blockade regime. In both cases, we argue that the results are qualitatively modified from the conventional ones if the insulating tunnel barrier is fractionalized. These experiments directly detect the possible existence of the chargon—a charge  $e$  spinless boson—in the insulator. The effects described in this paper provide a means to probing whether the undoped cuprate (despite its magnetism) is fractionalized. Thus, the experiments discussed here are complementary to the flux-trapping experiment we proposed in our earlier work [T. Senthil and Matthew P. A. Fisher, Phys. Rev. Lett. **86**, 292 (2001)].

DOI: 10.1103/PhysRevB.64.214511

PACS number(s): 75.10.Nr, 05.50.+q, 75.10.Jm

### I. INTRODUCTION

Superconductivity occurs in solids when a charged excitation with Bose statistics condenses. The electrons in a solid are fermions and cannot directly condense to produce superconductivity. A well-known solution to this difficulty is to pair electrons together into Cooper pairs. The Cooper pairs being bosons can then condense, giving rise to superconductivity. An alternative solution is to splinter the electron<sup>1-4</sup> into two pieces, thereby liberating its charge from its Fermi statistics. The resulting charged boson can then condense, leading to superconductivity.<sup>2,4,5</sup> Remarkably, the superconductor so obtained is in the same phase<sup>5-7</sup> as a BCS superconductor obtained by condensing Cooper pairs. In other words, both routes to superconductivity lead to the same final destination. In conventional metals, the occurrence of superconductivity is attributed to the presence of Cooper pairing of the Landau quasiparticles of the normal Fermi liquid state due to attractive interactions arising from phonons. In the cuprate high- $T_c$  superconductors, on the other hand, it may well be that the superconductivity occurs via the splintering of the electron. Some evidence for this is provided by angle-resolved photoemission experiments<sup>8</sup> which do not see any evidence for Landau quasiparticles in the normal state.

The quantization of electromagnetic flux in units of  $hc/2e$  is usually taken as evidence of the presence of Cooper pairing in superconductors. However, in the “nonpairing” fractionalization route to superconductivity—which is driven by condensation of a charge  $e$  chargon— $hc/2e$  flux quantization is nevertheless possible due to the presence of topological “vortexlike” excitations<sup>5,6,9</sup> in the normal state—dubbed the visons. Indeed, the existence of visons as gapped excitations is crucial<sup>5</sup> for the electron to be able to fractionalize at all. Recently, we proposed an experiment<sup>10,11</sup> that could directly detect the visons in the normal state, thereby providing a direct test of the idea that the electron fractionalizes in the cuprates.

In this paper, we propose several other tests of the idea that the electron is fractionalized in the nonsuperconducting state. These explore parts of the cuprate phase diagram that

are different from those explored by the vison detection experiment.

We first examine the Josephson effect in superconductor-insulator-superconductor junctions. In the classic ac Josephson effect, a dc voltage  $V$  applied to this junction leads to oscillations of the current at a frequency  $\omega = 2eV/\hbar$ . This fundamental result has been used to set the standard measure of the unit of voltage.<sup>12</sup> The factor of 2 indicates that the tunneling current is carried by charge  $2e$  Cooper pairs. In contrast to this classic effect, we argue that if the insulator in the junction is fractionalized, there will in addition be oscillations at a frequency  $\omega = eV/\hbar$ . The ratio of the amplitudes of the oscillations at the two different frequencies depends on the charge gap and the vison gap in the fractionalized insulator. A good candidate to maximize the amplitude of the  $eV/\hbar$  oscillation is the *undoped* cuprate insulator. The undoped cuprates are antiferromagnetic Mott insulators. However, as pointed out in Refs. 13 and 11, the fractionalization of the electron could coexist with the magnetism. Observation of  $eV/\hbar$  oscillations will establish experimentally the fractionalization of the electron in the undoped cuprate.

In recent years, a number of experiments<sup>14</sup> probing tunneling into small grains of conventional low- $T_c$  materials have shown an “even-odd” effect: the tunneling conductance has a periodic sequence of peaks as a function of the total charge on the grain. The period is *twice* the electron charge—this can be interpreted as due to Cooper pairing of electrons in the superconductor. We argue that this result will be modified if the insulating barrier in the tunnel junction is fractionalized. Specifically, we consider the situation where the tunneling occurs from superconducting leads through an insulating tunnel barrier to a small superconducting grain. If the insulator is fractionalized, it becomes possible for chargs to tunnel through. Consequently, the tunneling conductance would have a periodic sequence of peaks with period set by  $e$  rather than  $2e$ .

Another test of the fractionalization scenario for the underdoped cuprates was pointed out a long time ago by Sachdev<sup>7</sup> and Nagaosa and Lee.<sup>7</sup> They observed that a superconductor that descends from a fractionalized insulator has regimes in which the energy cost of an  $hc/e$  vortex is smaller than two isolated  $hc/2e$  vortices. Thus observation of

stable  $hc/e$  vortices in the superconducting phase would be an indirect test of the fractionalization in the “normal” state. Here, following their ideas and using currently available data, we provide a rough estimate of the region of stability of the  $hc/e$  vortex.

## II. $Z_2$ GAUGE THEORY

We begin by very briefly reviewing the theory of the fractionalized insulator. The excitations in the fractionalized insulator are a charge  $e$  spinless boson (the chargon) and a chargeless spin-1/2 fermion (the spinon). In addition, there is a gapped  $Z_2$  vortex excitation (the vison). The details of the spin physics in this fractionalized insulator are not important for our purposes: in particular, the insulator could have magnetic long-range order. In the context of the cuprates, this is significant. The undoped insulator certainly has Néel magnetic order, but may nevertheless also be fractionalized.<sup>13,11</sup>

A very convenient theoretical language to describe the fractionalized insulator is provided by the  $Z_2$  gauge theory formulation developed in Ref. 5. The  $Z_2$  formulation recasts a general class of interacting electron models as a theory of chargons and spinons minimally coupled to a fluctuating  $Z_2$  gauge field. A Hamiltonian version of the  $Z_2$  gauge theory is

$$H = H_c + H_\sigma + H_s, \quad (1)$$

$$H_c = - \sum_{\langle rr' \rangle} t_{rr'} \sigma_{rr'}^z (b_r^\dagger b_{r'} + \text{H.c.}) + U \sum_r (N_r - 1)^2, \quad (2)$$

$$H_\sigma = -K \sum_{\square} \prod_{\square} \sigma_{rr'}^z - h \sum_{\langle rr' \rangle} \sigma_{rr'}^x, \quad (3)$$

$$H_s = - \sum_{\langle rr' \rangle} \sigma_{rr'}^z [t_s (f_r^\dagger f_{r'} + \text{H.c.}) + \Delta_{rr'} (f_{r\uparrow} f_{r'\downarrow} - f_{r\downarrow} f_{r'\uparrow} + \text{H.c.})] + H_{int}[f]. \quad (4)$$

Here  $b_r^\dagger$  creates a chargon at site  $r$  while  $f_{r\alpha}$  creates a spinon with spin  $\alpha = \uparrow, \downarrow$  at site  $r$ . The operator  $N_r = b_r^\dagger b_r$  measures the number of bosons at site  $r$ . For simplicity, we have specialized to half-filling, i.e., to an average of one boson per site. The constant  $\Delta_{rr'}$  contains information about the pairing symmetry of the spinons. For the present, we assume that  $\Delta_{rr'}$  has  $d_{x^2-y^2}$  symmetry. The term  $H_{int}$  in the spinon Hamiltonian represents four spinon interaction terms which could induce antiferromagnetic ordering of the spin. The  $\sigma_{rr'}^z, \sigma_{rr'}^x$  are Pauli spin matrices that are defined on the links of the lattice. The  $\sigma_{rr'}^z$  may be thought of as  $Z_2$  gauge fields. The full Hamiltonian is invariant under the  $Z_2$  gauge transformation  $b_r \rightarrow -b_r, f_r \rightarrow -f_r$  at any site  $r$  of the lattice accompanied by letting  $\sigma_{rr'}^z \rightarrow -\sigma_{rr'}^z$  on all the links connected to that site. This Hamiltonian must be supplemented with the constraint equation

$$G_r = \prod_{r' \in r} \sigma_{rr'}^x e^{i\pi(f_r^\dagger f_{r'} + N_r)} = 1. \quad (5)$$

Here the product over  $\sigma_{rr'}^x$  is over all links that emanate from site  $r$ . The operator  $G_r$ , which commutes with the full

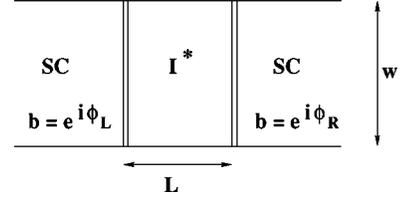


FIG. 1. Schematic of the superconductor-insulator-superconductor junction. Here  $I^*$  refers to the fractionalized insulator.  $\phi_L$  and  $\phi_R$  are the phases of the chargons in the left and right superconductors.

Hamiltonian, is the generator of the local  $Z_2$  gauge symmetry. Thus the constraint  $G_r = 1$  simply expresses the condition that the physical states in the Hilbert space are those that are gauge invariant.

The fractionalized insulating phase is described as the deconfined phase of this gauge theory. This is obtained when  $K \gg h, U \gg t_{rr'}$ . On the other hand, the conventional superconductor is described as a phase in which the chargons have condensed. This is obtained when  $t_{rr'} \gg U$  or, alternately, by doping away from half-filling. Note that the “pairing” symmetry of the superconductor is determined by  $\Delta_{rr'}$ .

## III. JOSEPHSON EFFECT

Now consider a superconductor-insulator-superconductor junction such as that shown in Fig. 1. We assume that the insulator is fractionalized. As the spin physics is irrelevant for the following discussion, we drop the spinon-dependent term in the Hamiltonian above and just focus on  $H_c + H_\sigma$ . We assume that  $t_{rr'} = t \gg U$  in both superconducting regions and  $t_{rr'} = t' \ll U$  in the insulating region. We also assume that  $K \gg h$  in the insulating region.

Inside the superconducting regions, we may safely ignore vortices in the phase of the chargon field. In particular, we may set  $\sigma_{rr'}^z = +1$  for all links inside the superconducting regions. The phase of the chargon field is fixed inside both superconducting regions. We write  $b_r \approx e^{i\phi_r}$  with  $\phi_r = \phi_L$  in the superconductor to the left and  $\phi_r = \phi_R$  in the superconductor to the right. We wish to derive the coupling between  $\phi_L$  and  $\phi_R$  due to the insulating region between the two superconductors. As  $t' \ll U$  in the fractionalized insulator, we may perturbatively integrate out the chargon degrees of freedom. For an insulating region of size  $L$  (measured in units of the lattice spacing), the lowest-order term coupling  $\phi_L$  and  $\phi_R$  is obtained in the  $L$ th order of perturbation theory. The resulting effective Hamiltonian is

$$H_{eff} = -t_{eff} \left( \sum_C \prod_C \sigma_{rr'}^z \right) \cos(\phi_L - \phi_R) + H_\sigma, \quad (6)$$

with  $t_{eff} \sim t' (t'/U)^{L-1}$ . Here  $C$  denotes a straight line path from any point on the left interface to the corresponding point on the right interface. To obtain the Josephson coupling, we further need to integrate out the  $Z_2$  gauge degrees of freedom. Consider the first term of the effective Hamiltonian  $H_{eff}$  above as a perturbation to  $H_\sigma$ . To leading order, we may replace  $\prod_C \sigma_{rr'}^z$  by its average evaluated with  $H_\sigma$ .

This average is readily found for small  $h/K$  (which is the appropriate limit in the fractionalized insulator). In the limit that  $h=0$ , we may set  $\sigma_{rr'}^z=1$ . For small  $h/K$ , each  $\sigma_{rr'}^z$  has an amplitude  $h$  to be negative while the energy cost for this fluctuation is of order  $K$ . Thus the average value of the product of  $\sigma_{rr'}^z$  over any path  $C$  will decay exponentially with the length of the path:  $\langle \prod_C \sigma_{rr'}^z \rangle \sim e^{-\lambda_v L}$  with  $\lambda_v \sim h/K$ . Thus, to leading order, we get the coupling

$$E_J^{(1)} = -t_1 \cos(\phi_L - \phi_R). \quad (7)$$

Here  $t_1 \sim U w e^{-(\lambda_c + \lambda_v)L}$  with  $\lambda_c = \ln(U/t')$ , and  $w$  is the lateral width of the junction. (Strictly speaking, this result assumes the thermodynamic limit in the lateral direction, i.e., large  $w$ —see below.)

Physically, this represents a direct coupling between the phases of the chargons in the two superconductors due to coherent tunneling of *chargons* through the intervening insulator. It is potentially also important to include the effect of coherent tunneling of *Cooper pairs* between the two superconductors. This is obtained at order  $2L$  in the perturbation theory when integrating out the chargin fields. The result is

$$E_J^{(2)} = -t_2 \cos(2\phi_L - 2\phi_R), \quad (8)$$

with  $t_2 \sim w t' (t'/U)^{2L-1} = w U e^{-2\lambda_c L}$ . Thus the full Josephson coupling between the two superconductors is

$$E_J = -t_1 \cos(\phi_L - \phi_R) - t_2 \cos(2\phi_L - 2\phi_R). \quad (9)$$

We emphasize that  $\phi_L, \phi_R$  represent the chargin phase in the two superconductors. The Cooper pair phase is twice the chargin phase. The second term is therefore the ‘‘standard’’ Josephson coupling while the first is novel, and arises due to the possibility of coherent tunneling of chargons through the fractionalized insulator. The ratio between the amplitudes of the chargin and Cooper pair tunneling terms is

$$\frac{t_1}{t_2} \sim e^{(\lambda_c - \lambda_v)L}. \quad (10)$$

Deep inside the fractionalized insulating phase, we have  $\lambda_c \sim \ln(U/t') > 1$  and  $\lambda_v \sim h/K \ll 1$ . Thus  $t_1$  will then dominate over  $t_2$ . In general, the optimal situation to maximize the ratio  $t_1/t_2$  is to have an insulating barrier with a large charge gap (i.e., large  $U/t'$ ) and a large vison gap (i.e., large  $K/h$ ).

In the context of the cuprates, this suggests that the best prospects for observing coherent chargin tunneling will occur if the insulating barrier is made of the *undoped* cuprate. This is as deeply insulating as is possible in the cuprates. Further, the vison gap (estimated to be of the order of the pseudogap temperature) is perhaps the largest in the undoped material.

From now on, we assume that  $t_1 \gg t_2$ . The form of the Josephson coupling with only the chargin tunneling term has the immediate consequence that the ac Josephson frequency will be  $eV/\hbar$ , which differs by a factor of 2 from the conventional one. This is a direct probe of the charge of the boson that tunnels coherently between the two superconductors. Observation of such  $eV/\hbar$  oscillations in the ac Joseph-

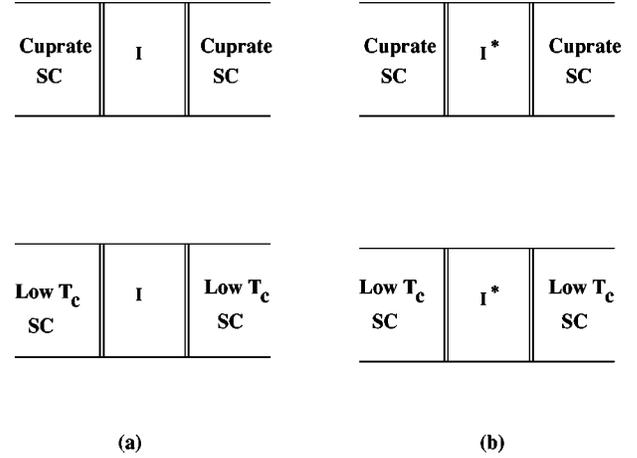


FIG. 2. Various kinds of Josephson junctions. In both cases in (a), the insulator  $I$  is not fractionalized. Then, the Josephson coupling occurs through Cooper pair tunneling and the ac Josephson frequency is  $2eV/\hbar$ . In both cases in (b), the insulator  $I^*$  is fractionalized. It is now possible for chargons to tunnel coherently between the two superconductors (independent of whether they are cuprate or low- $T_c$  superconductors), and the ac Josephson frequency will be  $eV/\hbar$ .

son effect will prove that the undoped cuprate is fractionalized (despite its Néel long-range order).

In passing, we note that even in the conventional ac Josephson effect as probed by the Shapiro steps in an irradiated junction, for instance, subharmonic oscillations at frequency  $2eV/n\hbar$  with integer  $n$  are present.<sup>12</sup> These occur due to the possibility of the absorption of  $n$  photons by the tunneling Cooper pairs. However, for small intensity of the radiation, the amplitude for the processes with  $n > 1$  is substantially smaller than for  $n = 1$ . We may then safely ignore the possibility of multiphoton absorption. As the intensity of the radiation is increased on the fractionalized junction, multiphoton absorption should also become possible, leading to subharmonic oscillations at frequencies  $eV/n\hbar$ .

It is important to note that our result does *not* depend on the ‘‘pairing’’ symmetry of the superconductor. In particular, the two superconductors could be conventional low- $T_c$   $s$ -wave materials (see Fig. 2). This perhaps surprising fact further emphasizes our point that Cooper pair condensation and chargin condensation lead to the same superconducting phase. The charge has no integrity as a good quantum number inside the superconducting state. Thus it is possible to halve the ac Josephson frequency to  $eV/\hbar$  by making an insulating barrier in which the chargons can freely propagate.

These general points are further illustrated by considering an insulating barrier made of a conventional material in which the electron is *not* fractionalized. Now chargons can no longer propagate coherently from one superconductor to the other. However, the Cooper pair tunneling can proceed as before. Formally, this may be seen in the  $Z_2$  formulation by noticing that the nonfractionalized phases are obtained when  $h \gg K$ . In this case, the operator  $\prod_C \sigma_{rr'}^z$  will fluctuate very rapidly with average value zero. The amplitude for single-chargin tunneling is thus zero, and only Cooper pair tunneling occurs. We therefore then obtain the standard ac Joseph-

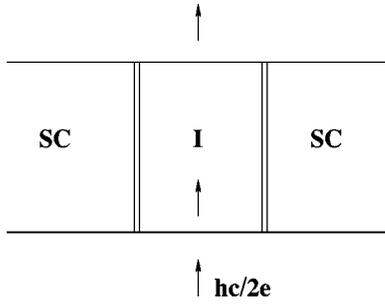


FIG. 3. The Josephson coupling in a conventional junction may be understood as due to slippage of  $hc/2e$  vortices through the insulator.

son effect with frequency  $2eV/\hbar$ . Once again, this result too holds independently of the pairing symmetry of the superconductor. In particular, this is true despite our description of the cuprate superconductors as a charge  $e$  condensate.

Further insight may be obtained by noting that the conventional Josephson effect may be thought of as due to a phase-slip process in which  $hc/2e$  vortices pass through the insulator as shown in Fig. 3. If the insulator is fractionalized, then it allows free propagation of  $hc/e$  vortices.<sup>13,5</sup> But the  $hc/2e$  vortices are gapped, and their propagation is suppressed. Indeed, the vison is precisely the remnant of the gapped  $hc/2e$  vortex in the fractionalized insulator.<sup>5</sup> Motion of an  $hc/e$  vortex across the junction (see Fig. 4) corresponds to a chargon phase slip by  $2\pi$  or, equivalently, a Cooper pair phase slip by  $4\pi$ . This then leads to ac Josephson oscillations at frequency  $eV/\hbar$ . If the lateral width  $w$  of the junction is finite, then the visons will slip through the interface at a rate that is exponentially small in  $w$ . This will then restore the conventional Josephson coupling at long time scales. Thus, the result in Eq. (9) assumes the limit of large  $w$  as mentioned earlier.

Another consequence of the possibility of coherent chargon tunneling through the insulator is that if a dc superconducting quantum interference device (SQUID) is made with fractionalized insulators for the barriers, the current will be a periodic function of the flux enclosed with period  $hc/e$  rather than the conventional  $hc/2e$ .

Some words of caution are necessary in performing experiments to look for the anomalous ac Josephson effect in

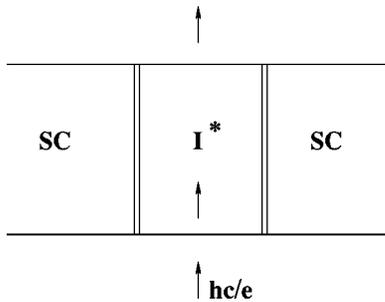


FIG. 4. The Josephson coupling with a fractionalized insulating barrier may be understood as due to slippage of  $hc/e$  vortices through the fractionalized insulator. The  $hc/2e$  vortices are not free to slip through the fractionalized insulator.

the undoped cuprate. First, if the cuprates are fractionalized, it seems most likely that the fractions of the electron are confined along the  $c$  axis. In particular, the chargons cannot tunnel coherently between successive copper-oxygen layers. This implies that the anomalous ac Josephson effect will be seen only in the geometry in which the  $c$  axis of the cuprate insulator is parallel to the interface. Second, thus far we have assumed a perfect interface between the superconductors and the insulator. If the interface is weak, so that the insulator is only weakly connected to one or both superconductors, the anomalous Josephson effect will not be seen. This is because, as argued in Ref. 11, along any line of weak contact, there will be a ( $T=0$ ) phase transition at which coherent chargon (or spinon) tunneling across the line will be blocked. Equivalently, along such a “weak line,” visons can slip through unhindered—this will restore the standard Josephson effect. It is therefore necessary to have interfaces that are good enough that vison slippage through the interface is prevented as it is elsewhere in the junction.

#### IV. COULOMB BLOCKADE (“DUAL” LITTLE-PARKS EXPERIMENT)

The hallmark of a superconductor is fluxoid quantization, which follows directly from the condensation of a charged boson. In a neutral superfluid, it is simply the vortex which is quantized. On the other hand, insulating states are characterized by a quantization of the electric charge. A direct way to measure this quantization is by exploiting the Coulomb blockade effect. In a typical geometry a small metallic “grain” is electrically isolated from two metallic leads by the presence of two insulating tunnel barriers. Upon tuning the voltage on a gate electrode  $V_g$ , conveniently located to capacitively couple into the metallic grain with capacitance  $C$ , it is possible to “charge up” the grain one electron at a time. This single-electron charging can be detected by measuring the electrical conductance through the grain as a function of the gate voltage. One finds a periodic sequence of conductance peaks with spacing  $\delta V_g = e/C$ —each peak occurring when there is a degeneracy between having  $n$  and  $n+1$  electrons on the grain. This Coulomb blockade experiment can be correctly thought of as the “dual” of the classic Little-Parks experiment—under the interchange of flux with charge.

Exploiting the Coulomb blockade to detect possible electron fractionalization in the underdoped cuprates is problematic since the chargon fragment carries the full electron charge. But as we now discuss, it should nevertheless be possible if the small metallic grain is replaced by a small superconducting grain. Coulomb blockade experiments involving a small superconducting grain connected to metallic leads via two insulating tunnel barriers have revealed<sup>14</sup> an astonishing “even-odd” effect. Due to the singlet pairing of electrons on the superconducting grain, adding an extra electron to a grain with an *odd* number of electrons is slightly less costly (the gap energy) than when the grain has an even number of electrons. This leads to an observable even-odd effect in the spacing between successive conductance peaks, with the period set by the distance between two peaks:  $\delta V_g$

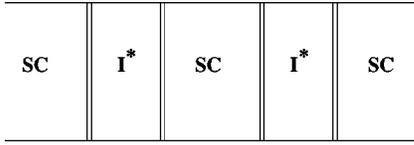


FIG. 5. Schematic of the Coulomb blockade experiment proposed to detect the chargon in the fractionalized insulator. The superconducting island in the middle is separated by fractionalized insulating tunnel barriers from the two superconducting leads. All junctions are assumed to be perfect.

$=2e/C$ —a charge- $2e$  periodicity corresponding to Cooper pairs of electron.

To detect the chargon, we propose redoing this Coulomb blockade experiment, making the insulating barriers from undoped cuprate material. Specifically, imagine a small superconducting grain which is electrically isolated from two superconducting leads with tunnel barriers made from undoped cuprate—as depicted schematically in Fig. 5. The ideal experiment involves using conventional  $s$ -wave low- $T_c$  superconductors.<sup>15</sup> We presume that with opaque barriers and a small grain, there is no Josephson coupling between the two electrodes. Nevertheless, each of the two barriers can be viewed as a Josephson junction connecting the grain to the external leads, with a Josephson coupling energy of the general form given in Eq. (9). If the undoped cuprates are deep within a fractionalized phase, the single-charge hopping term proportional to  $t_1$  will dominate the pair tunneling term. In that case, as one tunes a gate potential which is capacitively coupled to the grain, it should be a chargon which is discretely hopping onto the grain—not an electron or a Cooper pair. Since the superconducting grain is a chargon condensate (even for a low- $T_c$  material), these chargons can be readily absorbed by the condensate. This implies that the *conductance peaks should be charge- $e$  periodic*— $\delta V_g = e/C$ —with no even-odd effect present. As in the discussion of the Josephson effect, here too it is necessary to have very good interfaces so that single chargons can move freely through.

The halving of the charge- $2e$  periodicity when the insulating barriers are made from a fractionalized insulator, is indicative of a “vortex pairing.”<sup>13,5</sup> Specifically, a fractionalized insulator descends from a conventional superconductor when two  $hc/2e$  vortices pair and condense. The resulting  $hc/e$  vortex-pair condensate leads directly to halving of the charge- $2e$  periodicity on the superconducting grain. This is the dual analog to the halving of the electron flux quantization in the original Little-Parks experiment.

### V. STABLE $hc/e$ VORTICES

Several years ago, Sachdev<sup>7</sup> and Nagaosa and Lee<sup>7</sup> pointed out the possible stability of  $hc/e$  vortices in the superconducting state close to the transition to a spin-charge-separated normal phase. A single  $hc/2e$  vortex is still a stable object, but a pair of them have higher energy than an  $hc/e$  vortex. Here, we review the physics behind this observation, and use the currently available data to estimate the region of stability of the  $hc/e$  vortex.

The energy of any vortex in the superconducting state has two contributions. First there is the energy of the superflow. This is determined by the superfluid stiffness  $J$ —the coefficient of the  $(\nabla\varphi)^2/2$  term in the Landau-Ginzburg free energy for the phase of the Cooper pair. For a vortex of strength  $nhc/2e$ , the superflow energy is given by

$$E_{sf} = \pi n^2 J \ln\left(\frac{\lambda}{\xi}\right), \quad (11)$$

with  $\lambda$  and  $\xi$  being the penetration depth and the coherence lengths, respectively. The second contribution is the energy in the core of the vortex. If the underdoped cuprates emerge from a normal state that is fractionalized, the  $hc/2e$  vortex is made possible by the presence of the vison. Thus, the core energy of an  $hc/2e$  vortex includes the energy cost of a vison. Now let us assume that the very underdoped insulator is fractionalized. Then the vison gap is nonzero in the insulator and is expected to be smooth across the superconductor-insulator transition. Consequently, the core energy of the  $hc/2e$  vortex inside the superconducting state in the very underdoped regime is roughly the same as the vison gap and is nonzero on approaching the quantum transition to the insulator. On the other hand, the  $hc/e$  vortex does not have a vison in its core and its core energy vanishes on approaching the superconductor-insulator transition, assuming it is second order. (This is also consistent with the idea that the fractionalized insulator may be viewed as a condensate of  $hc/e$  vortices.) In our earlier work,<sup>4</sup> we have suggested that the vison gap is roughly of the order  $k_B T^*$  where  $T^*$  is the temperature associated with the pseudogap crossover. We may therefore estimate

$$E_{hc/2e}^{core} \approx k_B T^*. \quad (12)$$

It is clear from the above that the difference between the energy of a single  $hc/e$  vortex and that of two well-separated  $hc/2e$  vortices is

$$E_{hc/e}^{core} + 2\pi J \ln\left(\frac{\lambda}{\xi}\right) - 2k_B T^*. \quad (13)$$

If the  $T=0$  transition from the superconductor to the fractionalized insulator is second order, then the core energy of the  $hc/e$  vortex must vanish on approaching the transition. Further, we expect that this core energy will essentially be set by  $J$ —thus it is numerically smaller than the superflow energy by a factor of the order of  $\ln(\lambda/\xi) \approx 5$  (see below). For a rough estimate we drop it completely. Thus, for the  $hc/e$  vortex to be cheaper, we need

$$\pi J \ln\left(\frac{\lambda}{\xi}\right) \approx k_B T^*. \quad (14)$$

Clearly, this will always happen close enough to the transition. Empirically, the zero-temperature stiffness is proportional to  $k_B T_c$  in the underdoped regime. In YBCO, we have<sup>16</sup>  $J(T=0) \approx 1.4k_B T_c$ . Further, we have  $\lambda \approx 1600 \text{ \AA}$ ,  $\xi \approx 10 \text{ \AA}$ . We thus have the rough condition

$$7\pi T_c \approx T^* \quad (15)$$

on the maximum  $T_c$  for  $hc/e$  vortex stability.

The estimate above addresses the issue of the stability of the  $hc/e$  vortex at zero temperature. On moving up in temperature, the stiffness  $J$  decreases, thereby decreasing the superflow contribution, while there should be no significant change in the core energies. Thus, the  $hc/e$  vortex would gain in stability.

Considerable caution is required in trying to observe these stable  $hc/e$  vortices in experiments. The force between two  $hc/2e$  vortices is always repulsive at large separation (much bigger than the core size) where it is dominated by the superflow. Thus it is necessary for two well-separated  $hc/2e$  vortices to overcome the superflow energy barrier and get close enough before the gain in core energy of the  $hc/e$  vortex can provide for the attraction to bind them together. In practice, depending on the dynamics and the history of the sample, it may be possible for  $hc/2e$  vortices to be observable in some highly metastable state even in a regime in which a single  $hc/e$  vortex has lower energy than a pair of  $hc/2e$  ones.

## VI. DISCUSSION

In this paper, we have proposed a number of tests of the idea that the electron is fractionalized in the underdoped and undoped insulating cuprates. These experiments are sensitive to the presence of the chargon in the excitation spectrum of the insulator. As such, they allow for a direct experimental probe of the question of whether the undoped cuprate is fractionalized. In our earlier work,<sup>10,11</sup> we proposed an experiment to detect the vison in the underdoped cuprates. This vison detection experiment is, however, not very suitable to

addressing the issue of fractionalization in the undoped cuprate. The experiments proposed in this paper are thus complementary to this vison detection experiment. Taken together, we believe that a positive result in any of these experiments would be compelling evidence for fractionalization of the electron in the cuprates.

We conclude by reemphasizing one intriguing aspect of the results in this paper. We have argued that in both the ac Josephson effect and in the Coulomb blockade tunneling experiment, the outcome depends sensitively on whether the insulating tunnel barrier is fractionalized or not. Surprisingly, the precise nature (“pairing” symmetry) of the superconducting state is unimportant. In particular, both experiments should be possible with conventional low- $T_c$  superconductors. This implies that it is not always convenient to view even a low- $T_c$  superconductor as a Cooper pair condensate. Rather, for the experiments described here, the superconductor is best thought of as a *chargon* condensate. This ambiguity in the “charge of the condensate” is due to the fact that the charge is not a good quantum number in a superconductor, so that Cooper pairs (or chargons) do not *really* exist inside any superconductor.

## ACKNOWLEDGMENTS

We are particularly grateful to Kathryn Moler for her insightful questions which led to several of the results of this paper. We also thank L. Balents, J. Berlinsky, D. Drew, C. Kallin, P.A. Lee, T. Leggett, and S. Sachdev for useful discussions. This research was supported by the NSF under Grant Nos. DMR-97-04005, DMR95-28578, and PHY99-07949.

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