Physics 219 Problem Set 4 (due 2/10)

Problem 1 (30)

Construct the normalized canonical distribution function for a single particle in a uniform gravitational field in a cylinder of crossection area A whose axis is aligned along the field. Calculate the probability of the particle to be between h and $h + dh$ above the base, as well as the mean hight and mean energy

Problem 2 (30)

Derive Stirling's Formula for $lnN!$ starting from the exact expression

$$N! = \int_0^\infty x^N e^{-x} dx \tag{1}$$

by the method of steepest descents. (Hint: Rewrite the integral in the form $\int_0^\infty e^{Ng(x)} dx$ expand $g(x)$ about its maximum x_0 keeping terms up to order $(x - x_0)^2$. Why is this a good approximation for large N? Give an argument for the form of next correction.

Problem 3 (40)

The method of generating functions or "discrete Laplace transforms" has applications besides converting a canonical partition function into a grand canonical one! Use it to find a general expression for the Fibbonacci numbers

$$\{f_n\} = \{0, 1, 2, 3, 5, 8, 13, 21, ...\} \tag{2}$$

with $f_0 = 0$ and $f_1 = 1$. First, find a three term recursion relation for f_n. Take a discrete Laplace transform of this relation and solve for the generating function

$$F(z) = \sum_{n=0}^{\infty} z^n f_n \tag{3}$$

Separate $F(z)$ into partial fractions and expand each fraction in powers of z. Thus find an explicit expression for f_n and state its behavior for larg n.