Physics 219 Problem Set 4 (due 2/10)

Problem 1 (30)

Construct the normalized canonical distribution function for a single particle in a uniform gravitational field in a cylinder of crosssection area $A$ whose axis is aligned along the field. Calculate the probability of the particle to be between $h$ and $h + dh$ above the base, as well as the mean height and mean energy.

Problem 2 (30)

Derive Stirling’s Formula for $\ln N!$ starting from the exact expression

$$N! = \int_0^\infty x^N e^{-x} dx$$

(1)

by the method of steepest descents. (Hint: Rewrite the integral in the form $\int_0^\infty e^{Ng(x)} dx$ expand $g(x)$ about its maximum $x_0$ keeping terms up to order $(x - x_0)^2$. Why is this a good approximation for large $N$? Give an argument for the form of next correction.

Problem 3 (40)

The method of generating functions or “discrete Laplace transforms” has applications besides converting a canonical partition function into a grand canonical one! Use it to find a general expression for the Fibonacci numbers

$$\{f_n\} = \{0, 1, 2, 3, 5, 8, 13, 21, \ldots\}$$

(2)

with $f_0 = 0$ and $f_1 = 1$. First, find a three term recursion relation for $f_n$. Take a discrete Laplace transform of this relation and solve for the generating function

$$F(z) = \sum_{n=0}^{\infty} z^n f_n$$

(3)

Separate $F(z)$ into partial fractions and expand each fraction in powers of $z$. Thus find an explicit expression for $f_n$ and state its behavior for large $n$. 