Problem Set 6 (due 2/24).

Problem 1 (20). Derive the Van der Waals equation of state for imperfect gas starting with the following phenomenological equations for internal energy

$$E = N\left[\frac{3}{2}k_B T - \frac{aN}{V}\right] \tag{1}$$

and entropy

$$S = N\log\left[\frac{V-bN}{N\Lambda^3}\right] \tag{2}$$

Sketch $pV + F(T,V,N)$, define Gibbs free energy $G(p,T,N) = \min_V\left[pV + F(T,V,N)\right]$ and identify the phase coexistence condition.

Problem 2. (30) The Debye theory treats the crystal as a continuum body and hence the dispersion relation for the phonons is $\omega = ck$ where c is the speed of sound. In a ferromagnetic solid at low temperatures there exist quantized waves of magnetization called "spin waves" that have the dispersion relation $\omega = Dk^2$. Find the low temperature heat capacity due to such spin waves.

Problem 3. (50) Consider a one dimensional gas of "hard rods" described by a Hamiltonian

$$H = \sum_{j=1}^{N}\frac{p_j^2}{2m} + \sum_{i\neq j}\phi(x_i - x_j) \tag{3}$$

where $\phi(y) = \infty$ if $|y| < a$ and $\phi(y) = 0$ if $|y| > a$

Calculate the classical canonical partition function:

a) Integrate out the momentum part of the phase space and define the configurational part of the partition function

b) Evaluate (exactly) the configurational partition function and the define the free energy of the 1D gas of rods.

c) Calculate the limiting free energy and the equation of state as N and L (system size) go to infinity while keeping conbstant density $\rho = N/L$. Hence construct the "virial" (i.e. low density) expansion for p.

HINT: Consider integration over a restricted region $x_0 < x_1 < x_2 < ... < x_{N+1}$ so that "atoms" are labeled in their order along the line and interactions involve only the nearest neighbors. How many equivalent such regions are there? Note that with that restriction the effective range of x_1 is $0 \leq x_1 \leq x_2 - a$ because of hard core repulsion. Similarly the x_2 integration runs over $a \leq x_2 \leq x_3 - a$, etc. Use this to do the relevant integrations in order. Your answer should look quite similar to that for an ideal 1D gas...