Problem Set 3: due 2/3

Problem 1 (30)

a) Use the canonical partition function to calculate the entropy of an ideal gas of $2N$ identical atoms confined in volume $2V$, held at temperature T, and compare with the entropy of an ideal gas consisting of a mixture of N atoms of "X" and N atoms of "Y" in the same volume $2V$. The difference of entropies for these two cases, is called the "entropy of mixing". What would be the entropy if all of the $2N$ atoms were different?

b) What would happen if gas components "X" and "Y" were initially separated in two adjacent compartments each of volume V, initially at T, and then connected and allowed to mix adiabatically (i.e. without transfer of heat in or out of the system)? What would happen upon the same procedure if compartments contained the same gas?

Problem 2 (30)

Find a set of probabilities p_i which maximize the entropy $S = -\sum_i p_i \, ln p_i$ while preserving the total energy constraint $\sum_i E_i p_i = U$ where E_i and U are fixed parameters.

Problem 3 (40)

Grand canonical partition function is defined by

$$Z_{GC}(T, V, \mu) \;=\; \sum_N e^{\beta\mu N - \beta F(T,V,N)} \tag{1}$$

and the GC ensemble average of N is

$$< N > = \; Z_{GC}^{-1} \sum_N N e^{\beta\mu N - \beta F(T,V,N)} \tag{2}$$

In a thermodynamic system the sum is dominated by large N and it is OK to approximate the sum by an integral.

a) Determine $< N >$ using saddle point approximation (following the approach similar to the calculation of $< E >$ for the canonical ensemble.

b) Derive an expression for $< \Delta N^2 >$.