27 the electric field

With every point in space near the earth we can associate a *gravitational field* vector \mathbf{g} (see Eq. 16-12). This is the gravitational acceleration that a test body, placed at that point and released, would experience. If *m* is the mass of the body and \mathbf{F} the gravitational force acting on it, \mathbf{g} is given by

27-1 *THE ELECTRIC FIELD*

$$\mathbf{g} = \mathbf{F}/m. \tag{27-1}$$

This is an example of a *vector field*. For points near the surface of the earth, the field is often taken as *uniform*; that is, \mathbf{g} is the same for all points.

The flow of water in a river provides another example of a vector field, called a *flow field* (see Section 18-7). Every point in the water has associated with it a vector quantity, the velocity \mathbf{v} with which the water flows past the point. If \mathbf{g} and \mathbf{v} do not change with time, the corresponding fields are described as *stationary*. In the case of the river note that, even though the water is moving, the vector \mathbf{v} at any point does not change with time for steady-flow conditions.

If we place a test charge in the space near a charged rod, an electrostatic force will act on the charge. We speak of an *electric field* in this space. In a similar way, we speak of a *magnetic field* in the space around a bar magnet. In the classical theory of electromagnetism the electric and magnetic fields are central concepts. In this chapter we deal with electric fields associated with charges viewed from a reference frame in which they are at rest, that is, with *electrostatics*.

Before Faraday's time, the force acting between charged particles was thought of as a direct and instantaneous interaction between the two particles. This *action-at-a-distance* view was also held for magnetic and for gravitational forces. Today we prefer to think in terms of electric

fields as follows:

- 1. Charge q_1 in Fig. 27-1 sets up an electric field in the space around itself. This field is suggested by the shading in the figure; later we shall show how to represent electric fields more concretely.
- 2. The field acts on charge q_2 ; this shows up in the force **F** that q_2 experiences.





The field plays an intermediary role in the forces between charges. There are two separate problems: (a) calculating the fields that are set up by given distributions of charge and (b) calculating the forces that given fields will exert on charges placed in them. We think in terms of

$$charge \rightleftharpoons field \rightleftharpoons charge$$

and not, as in the action-at-a-distance point of view, in terms of

$charge \rightleftharpoons charge$

In Fig. 27-1 we can also imagine that q_2 sets up a field and that this field acts on q_1 , producing a force – **F** on it in accord with Newton's third law. The situation is completely symmetrical, each charge being immersed in a field associated with the other charge.

If the only problem in electromagnetism were that of the forces between stationary charges, the field and the action-at-a-distance points of view would be perfectly equivalent. Suppose, however, that q_1 in Fig 27-1 suddenly accelerates to the right. How quickly does the charge q_2 learn that q_1 has moved and that the force which it (q_2) experiences must increase? Electromagnetic theory predicts that q_2 learns about q_1 's motion by a field disturbance that emanates from q_1 , traveling with the speed of light. The action-at-a-distance point of view requires that information about q_1 's acceleration be communicated instantaneously to q_2 ; this is not in accord with experiment.^{*} Accelerating electrons in the antenna of a radio transmitter influence electrons in a distant receiving antenna only after a time l/c where l is the separation of the antennas and c is the speed of light.

^{*}By introducing other considerations it is possible to develop a consistent program of electromagnetism from the action-at-a-distance point of view. This is not commonly done however and we will not do so in this book.

To define the electric field operationally, we place a small test charge q_0 (assumed positive for convenience) at the point in space that is to be examined, and we measure the electrical force **F** (if any) that acts on this body. The electric field **E** at the point is defined as^{*}

$$\mathbf{E} = \mathbf{F}/q_0.$$

Here **E** is a vector because **F** is one, q_0 being a scalar. The direction of **E** is the direction of **F**, that is, it is the direction in which a resting positive charge placed at the point would tend to move.

The definition of gravitational field g is much like that of electric field, except that the mass of the test body rather than its charge is the property of interest. Although the units of g are usually written as m/s2, they could also be written as N/kg (Eq. 27-1); those for E are N/C (Eq. 27-2). Thus both g and E are expressed as a force divided by a property (mass or charge) of the test body.

What is the magnitude of the electric field E such that an electron, placed in the field, would experience an electrical force equal to its weight?

From Eq. 27-2, replacing q_0 by *e* and **F** by *mg*, where *m* is the electron mass, we have

$$E = \frac{F}{q_0} = \frac{mg}{e}$$

= $\frac{(9.1 \times 10^{-31} \text{ kg})(9.8 \text{ m/s}^2)}{1.6 \times 10^{-19} \text{ C}}$
= $5.6 \times 10^{-11} \text{ N/C}$

This is a very weak electric field. Which way will **E** have to point if the electric force is to cancel the gravitational force?

In applying Eq. 27-2 we must use a test charge as small as possible. A large test charge might disturb the primary charges that are responsible for the field, thus changing the very quantity that we are trying to measure. Equation 27-2 should, strictly, be replaced by

$$\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}}{q_0} \tag{27-3}$$

This equation instructs us to use a smaller and smaller test charge q0, evaluating the ration \mathbf{F}/q_0 at every step. The electric field \mathbf{E} is then the limit of this ratio as the size of the test charge approaches zero.[†]

The concept of the electric field as a vector was not appreciated by Michael Faraday, who always thought in terms of *lines of force*. The lines of force still form a convenient way of visualizing electric-field patterns. We shall use them for this purpose but we shall not employ them quantitatelively.

The relationship between the (imaginary) lines of force and the electric field vector is this:

27-2 The electric field **E**

EXAMPLE 1

27-3 LINES OF FORCE

^{*} This definition of E, though conceptually sound and quite appropriate to our present purpose, is rarely carried out in practice because of experimental difficulties. E is normally found by calculation from more readily measurable quantities such as the electric potential; see Section 29-7.

[†] Of course, q_0 can never be less than the electronic charge *e*.

- 1. The tangent to a line of force at any point gives the *direction* of **E** at that point.
- 2. The lines of force are drawn so that the number of lines per unit cross-sectional area (perpendicular to the lines) is proportional to the magnitude of E. Where the lines are close together E is large and where they are far apart E is small.

It is not obvious that it is possible to draw a continuous set of lines to meet these requirements. Indeed, it turns out that if Coulomb's law were not true, it would *not* be possible to do so, see Problem 7.

Figure 27-2 shows the lines of force for a uniform sheet of positive charge. We assume that the sheet is infinitely large, which, for a sheet of finite dimensions, is equivalent to considering only those points whose distance from the sheet is small compared to the distance to the nearest edge of the sheet. A positive test charge, released in front of such a sheet, would move away from the sheet along a perpendicular line. Thus the electric field vector at any point near the sheet must be at right angles to the sheet. The lines of force are uniformly spaced, which means that **E** has the same magnitude for all points near the sheet.

Figure 27-3 shows the lines of force for a negatively charged sphere. From symmetry, the lines must lie along radii. They point inward because a free positive charge would be accelerated in this direction. The electric field E is not constant but decreases with increasing distance from the charge. This is evident in the lines of force, which are farther apart at greater distances. From symmetry, E is the same for all points that lie a given distance from the charge.







figure 27-3 Lines of force for a negatively charged sphere.

In Fig. 27-3 how does *E* vary with the distance *r* from the center of the charged sphere? Suppose that *N* lines terminate on the sphere. Draw an imaginary concentric sphere of radius *r*; the number of lines per unit cross-sectional area at every point on the sphere is $N/4\pi r^2$. Since *E* is proportional to this, we can write that

$$E \propto 1/r^2$$
.

We derive an exact relationship in Section 27-4. How does E vary with distance from an infinitely long uniform cylinder of charge?



Figures 27-4 and 27-5 show the lines of force for two equal like charges and for two equal unlike charges, respectively. Michael Faraday, as we have said, used lines of force a great deal in his thinking. They were more real for him than they are for most scientists and engineers today. It is possible to sympathize with Faraday's point of view. Can we not almost



figure 27-4 Lines of force for two equal positive charges.

figure 27-5 Lines of force for equal but opposite charges.

"see" the charges being pushed apart in Fig. 27-4 and pulled together in Fig. 27-5 by the lines of force? Compare Fig. 27-4 with Fig. 18-14, which represents a flow field. Figure 27-6 shows a representation of lines of force around charged conductors, using grass seeds suspended in an insulating liquid.

Lines of force give a vivid picture of the way **E** varies through a given region of space. However, the equations of electromagnetism (see Table 40-2) are written in terms of the electric field **E** and other field vectors and not in terms of the lines of force. The electric field **E** varies in a perfectly continuous way as any path in the field is traversed, see Fig. 27-7.



figure 27-6

Photographs of the patterns of electric lines of force around (a) a pair of charged plates [compare Fig. 27-2], and (b) two rods with equal and opposite charges [compare Fig. 27-5]. The patterns were made by suspending thread in an insulating liquid. (Courtesy NASA. <u>http://son.nasa.gov/tass/content/</u> <u>electricity.htm</u>).

figure 27-7 E varies continuously as we move along any path AB in the field set up by point charge +Q. In general, the path AB and the field vectors E will not lie in the plane of the figure. In this section we deal with the charge-field interaction by showing how we may calculate E for various points near given charge distributions, starting with the sample case of a point charge q.

Let a test charge q_0 be placed a distance r from a point charge q. The magnitude of the force acting on q_0 is given by Coulomb's law, or

$$F = \frac{1}{4\pi\varepsilon_o} \frac{qq_0}{r^2}$$

The electric field at the site of the test charge is given by Eq. 27-2, or

$$E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$
(27-4)

The direction of E is on a radial line from q, pointing outward if q is positive and inward if q is negative.

To find **E** for a group of point charges: (a) Calculate **E**, due to each charge at the given point *as if it were the only charge present*. (b) Add these separately calculated fields vectorially to find the resultant field **E** at the point. In equation form,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \dots = \sum \mathbf{E}_n \quad n = 1, 2, 3, \dots$$
 (27-5)

The sum is a vector sum, taken over all the charges. Equation 27-5 (like Eq. 26-4) is an example of the *principle of superposition* which states, in this context, that at a given point the electric fields due to separate charge distributions simply add up (vectorially) or superimpose independently. The principle of superposition is very important in physics. It applies, for example, to gravitational and magnetic situations as well.^{*}

If the charge distribution is a continuous one, the field it sets up at any point P can be computed by dividing the charge into infinitesimal elements dq. The field $d\mathbf{E}$ due to each element at the point in question is then calculated, treating the elements as point charges. The magnitude of $d\mathbf{E}$ (see Eq. 27-4) is given by

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2}$$
(27-6)

where r is the distance from the charge element dq to the point P. The resultant field at P is then found from the superposition principle by adding (that is, integrating) the field contributions due to all the charge elements, or,

$$\mathbf{E} = \int d\mathbf{E} \tag{27-7}$$

The integration, like the sum in Eq. 27-5 is a vector operation; in Example 5 we shall see how such an integral is handled in a simple case.

27-4 CALCULATION OF **E**

An electric dipole. Figure 27-8 shows a positive and a negative charge of equal magnitude q placed at a distance 2a apart, a configuration called an electric dipole. The pattern of lines of force is that of Fig. 27-5, which also shows an electric dipole. What is the field **E** due to these charges at point P a distance r along the perpendicular bisector of the line joining the charges? Assume r >> a.

^{*} Formally, the principle of superposition holds in physics only to the extent that the differential equation defining the situation is linear. To the extent that the amplitudes of mechanical or electromagnetic oscillations become relatively large, the principle tends to fail. We do not discuss such cases in this book. In particular, the principle holds absolutely in electrostatics.

Equation 27-5 gives the vector equation

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

where, from Eq. 27-4,*

$$E_1 = E_2 = \frac{1}{4\pi\varepsilon_0} \frac{q}{a^2 + r^2}$$

The vector sum of E_1 and E_2 points vertically downward and has the magnitude

$$E = 2E_1 \cos \theta$$

From the figure we see that

$$\cos\theta = \frac{a}{\sqrt{a^2 + r^2}}$$

Substituting the expressions for E_1 and for $\cos \theta$ into that for E yields

$$E = \frac{2}{4\pi\varepsilon_0} \frac{q}{(a^2 + r^2)} \frac{a}{\sqrt{a^2 + r^2}} = \frac{1}{4\pi\varepsilon_0} \frac{2aq}{(a^2 + r^2)^{3/2}}$$

If r >> a, we can neglect a in the denominator; this equation then reduces to

$$E \approx \frac{1}{4\pi\varepsilon_0} \frac{(2a)(q)}{r^3}.$$
 (27-8a)

The essential properties of the charge distribution in Fig. 27-8, the magnitude of the charge q and the separation 2a between the charges, enter Eq. 27-8a only as a product. This means that, if we measure E at various distances from the electric dipole (assuming r >> a), we can never deduce q and 2a separately but only the product 2aq; if q were doubled and a simultaneously cut in half, the electric field *at large distances from the dipole* would not change.

The product 2aq is called the *electric dipole moment p*. Thus we can rewrite this equation for *E*, for distant points along the perpendicular bisector, as

$$E \approx \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3}.$$
 (27-8b)

The result for distant points *along the dipole axis* (See Problem 25) and the general result for any distant point (see Problem 28) also contain the quantities 2a and q only as the product 2aq (= p). The variation of E with r in the general result for distant points is also as $1/r^3$, as in Eq. 27-8b.

The dipole of Fig. 27-8 is two equal and opposite charges placed close to each other so that their separate fields at distant points almost, but not quite, cancel. From this point of view it is easy to understand that E(r) for a dipole varies at large distances as $1/r^3$ (Eq. 27-8b), whereas for a point charge E(r) drops off more slowly, namely as $1/r^2$ (Eq. 27-4).

Figure 27-9 shows a charge q_1 (= +1.0 × 10⁻⁶ C) 10 cm from a charge q_2 (= +2.0 × 10⁻⁶ C). At what point on the line joining the two charges is the electric field zero?

The point must lie between the charges because only here do the forces exerted by q_1 and q_2 on a test charge (no matter whether it is positive or negative) oppose each other. If \mathbf{E}_1 is the electric field due to q_1 and \mathbf{E}_2 that due to q_2 , we must have

$$E_1 = E_2$$

or (see Eq. 27-4)

 $\frac{1}{4\pi\varepsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(l-x)^2}$

figure 27-8 Example 3.

* Note that the *r*'s in Eq. 27-4 and in this equation have different meanings.

where x is the distance from q_1 and l equals 10 cm. Solving for x yields

$$x = \frac{l}{1 + \sqrt{q_2 / q_1}} = \frac{10 \text{ cm}}{1 + \sqrt{2}} = 4.1 \text{ cm}.$$

Supply the missing steps. On what basis was the second root of the resulting quadratic equation discarded?

Ring of charge. Figure 27-10 shows a ring of charge q and radius a. Calculate **E** for points on the axis of the ring a distance x from its center.

Consider a differential element of the ring of length ds, located at the top of the ring in Fig. 27-10. It contains an element of charge given by

$$dq = q \frac{ds}{2\pi a}$$

where $2\pi a$ is the circumference of the ring. This element sets up a differential electric field $d\mathbf{E}$ at point *P*.

The resultant field \mathbf{E} and P is found by integrating the effects of all the elements that make up the ring. From symmetry this resultant field must lie along the ring axis. Thus only the component of $d\mathbf{E}$ parallel to this axis contributes to the final result. The component perpendicular to the axis is canceled out by and equal but opposite component established by the charge element on the opposite side of the ring.

Thus the general vector integral (Eq. 27-7)

$$\mathbf{E} = \int d\mathbf{E}$$

becomes a scalar integral $E = \int dE \cos \theta$.

The quantity dE follows from Eq. 27-6, or

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0} \left(\frac{q \, ds}{2\pi a}\right) \frac{1}{a^2 + x^2}$$

 $\cos\theta = \frac{x}{\sqrt{a^2 + x^2}}.$

From Fig. 27-10 we have

figure 27-10 Example 5.

Noting that, for a given point P, x has the same value for all the charge elements and is not a variable and that s is the variable of integration, we obtain

$$E = \int dE \cos\theta = \int \frac{1}{4\pi\varepsilon_0} \frac{q \, ds}{(2\pi a)(a^2 + x^2)} \frac{x}{\sqrt{a^2 + x^2}}$$
$$= \frac{1}{4\pi\varepsilon_0} \frac{qx}{(2\pi a)(a^2 + x^2)^{3/2}} \int ds.$$

The integral is simply the circumference of the ring (= $2\pi a$), so that

$$E = \frac{1}{4\pi\varepsilon_0} \frac{qx}{(a^2 + x^2)^{3/2}}.$$

Does this expression for *E* reduce to an expected results for x = 0? For x >> a we can neglect *a* in the denominator of this equation, yielding

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{x^2}$$

This is an expected result (compare Eq. 27-4) because at great enough distances the ring behaves like a point charge q.

Infinite line of charge. Figure 27-11 shows a section of an infinite line of charge whose linear charge density (that is, the charge per unit length, measured in C/m) has the constant value λ . Calculate the field **E** a distance *y* from the line.

The magnitude of the field contribution dE due to charge element $dq (= \lambda dx)$ is given, using Eq. 27-6, by

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda \, dx}{y^2 + x^2}$$

EXAMPLE 6

figure 27-11 Example 6. A section of an infinite line of charge

The vector dE, as Fig. 27-11 shows, has the components

$$dE_x = -dE\sin\theta$$
 and $dE_y = dE\cos\theta$

The minus sign in front of dE_x indicates the dE_x points in the negative x direction. The x and y components of the resultant vector **E** at point P are given by

$$E_x = \int dE_x = -\int_{x=-\infty}^{x=+\infty} \sin\theta \, dE$$
 and $E_y = \int dE_y = -\int_{x=-\infty}^{x=+\infty} \cos\theta \, dE$

 E_x must be zero because every charge on the right has a corresponding element on the left such that their field contributions in the *x* direction cancel. The **E** points entirely in the *y* direction. Because the contributions to E_y from the right- and left-hand halves of the rod are equal, we can write

$$E = E_y = 2 \int_{x=0}^{x=+\infty} \cos \theta \, dE$$

Note that we have changed the lower limit of integration and have introduced a compensating factor of two.

Substituting the expression for dE into this equation gives

$$E = \frac{\lambda}{2\pi\varepsilon_0} \int_{x=0}^{x=\infty} \cos\theta \, \frac{dx}{y^2 + x^2}$$

From Fig. 27-11 we see that the quantities θ and x are not independent. We must eliminate one of them, say x. The relation between x and θ is (see figure)

$$x = y \tan \theta$$
.

Differentiating, we obtain

 $dx = y \sec^2 \theta \, d\theta \, .$

Substituting these two expressions leads finally to

$$E = \frac{\lambda}{2\pi\varepsilon_0 y} \int_{\theta=0}^{\theta=\pi/2} \cos\theta \, d\theta.$$

You should check this step carefully, noting that the limits must now be on θ and not on x. For example, as $x \to +\infty$, $\theta \to \pi/2$, as Fig. 27-11 shows. This equation integrates readily to

$$E = \frac{\lambda}{2\pi\varepsilon_0 y} \cos\theta \Big|_0^{\pi/2} = \frac{\lambda}{2\pi\varepsilon_0 y}.$$

You may wonder about the usefulness of solving a problem involving an infinite rod of charge when any actual rod must have a finite length (see Problem 23). However, for points close enough to finite rods and not near their ends, the equation that we have just derived yields results that are so close to the correct values that the difference can be ignored in many practical situations. It is usually unnecessary to solve exactly every geometry encountered in practical problems. Indeed, if idealizations or approximations are not made, the vast majority of significant problems of all kinds in physics and engineering cannot be solved at all.

Here and in the following section, in contrast with Section 27-4, we investiage the other half of the charge-field interaction, namely, if we are given a field \mathbf{E} , what forces and torques will act on a charge configuration placed in it? We start with the simple case of a point charge in a uniform electric field.

An electric field will exert a force on a charged particle given by (Eq. 27-2)

$$\mathbf{F} = \mathbf{E}q$$
.

This force will produce an acceleration

$$\mathbf{a} = \mathbf{F}/m$$

where *m* is the mass of the particle. We will consider two examples of the acceleration of a charged particle in a uniform electric field. Such a field can be produced by connecting the terminals of a battery to two parallel metal plates that are otherwise insulated from each other. If the spacing between the plates is small compared with the dimensions of the plates, the field between them will be fairly uniform except near the edges. Note that in calculating the motion of a particle in a field set up by external charges, the field due to the particle itself (that is, its *self-field*) is ignored. In the same way, the earth's gravitational field can exert no net force on the earth itself but only on a second object, say a stone, placed in that field.

27-5

A POINT CHARGE IN AN ELECTRIC FIELD A particle of mass m and charge q is placed at rest in a uniform electric field (Fig. 27-12) and released. Describe its motion.

EXAMPLE 7

figure 27-12

Example 7. A charge released from rest in a uniform electric field set up between two oppositely charged metal plates P_1 and P_2 .

The motion resembles that of a body falling in the earth's gravitational field. The (constant) acceleration is given by

$$a = \frac{F}{m} = \frac{qE}{m}.$$

The equations for uniformly accelerated motion (Table 3-1) then apply. With v0 = 0, they are

$$v = at = \frac{qEt}{m},$$
$$y = \frac{1}{2}at^{2} = \frac{qEt^{2}}{2m},$$
$$v^{2} = 2ay = \frac{2qEy}{m}.$$

and

The kinetic energy attained after moving a distance *y* is found from

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{2qEy}{m}\right) = qEy.$$

This results also follows directly from the work-energy theorem because a constant force qE acts over a distance y.

Deflecting an electron beam. Figure 27-13 shows an electron of mass m and charge e projected with speed v0 at right angles to a uniform field **E**. Describe its motion.

EXAMPLE 8

figure 27-13 Example 8. An electron is projected into a uniform electric field.

The motion is like that of a projectile fired horizontally in the earth's gravitational field. The considerations of Section 4-3 apply, the horizontal (x) and vertical (y) motions being

given by

and

$$x = v_0 t$$
$$y = \frac{1}{2}at^2 = \frac{eE}{2m}t^2.$$

Eliminating t yields

$$y = \frac{eE}{2mv_0^2} x^2$$
(27-9)

for the equation of the trajectory.

When the electron emerges from the plates in Fig. 27-13, it travels (neglecting gravity) in a straight line tangent to the parabola of Eq. 27-9 at the exit point. We can let it fall on a fluorescent screen S placed some distance beyond the plates. Together with other electrons following the same path, it will then make itself visible as a small luminous spot, this is the principle of the electrostatic *cathode-ray oscilloscope*.

The electric field between the plates of a cathode-ray oscilloscope is 1.2×10^4 N/C. What deflection will an electron experience if it enters at right angles to the field with a kinetic energy of 2000 eV (= 3.2×10^{-16} J), a typical value? The deflecting assembly is 1.5 cm long.

Recalling that $K_0 = \frac{1}{2} m v_0^2$, we can rewrite Eq. 27-9 as

$$y = \frac{eE}{4K_0} x^2.$$

If x_1 is the horizontal position of the far edge of the plate, y_1 will be the corresponding deflection (see Fig. 27-13), or

$$y = \frac{eE}{4K_0} x_1^2$$

= $\frac{(1.6 \times 10^{-19} \text{ C})(1.2 \times 10^4 \text{ N/C})(1.5 \times 10^{-2} \text{ m})^2}{(4)(3.2 \times 10^{-16} \text{ J})}$
= $3.4 \times 10^{-4} \text{ m} = 0.34 \text{ mm}$

The deflection measured, not at the deflecting plates but at the fluorescent screen, is much larger.

A positive point test charge q_0 is placed halfway between two equal fixed positive charges q. What force acts on it at or near this point P?

From symmetry the force at the point is zero so that the particle is in equilibrium; the nature of the equilibrium remains to be found. Figure 27-14 (compare Fig. 27-4) shows the E vectors for four points near P. If the test charge is moved along the z axis, a *restoring* force is brought into play; however, the equilibrium is unstable for motion along the x (and y) axes. Thus we have the three-dimensional equivalent of *saddle point equilibrium*; see Fig. 14-8. What is the nature of the equilibrium for a negative test charge?

An electric dipole moment can be regarded as a vector \mathbf{p} whose magnitude, p, for a dipole like that described in Example 3, is the product 2aq of the magnitude of either charge q and the distance 2a between the charges. The *direction* of \mathbf{p} for such a dipole is from the *negative* to the *positive* charge. The vector nature of the electric dipole moment permits us to cast many expressions involving electric dipoles into concise form, as we shall see.

27-6 *A DIPOLE IN AN ELECTRIC FIELD*

figure 27-14 Example 10. The electric field at four points near a point *P* which is centered between two equal positive charges.

figure 27-15

(*a*) An electric dipole in a uniform external field. (*b*) Illustrating $\tau = \mathbf{p} \times \mathbf{E}$.

Figure 27-15*a* shows an electric dipole formed by placing two charges +q and -q a fixed distance 2a apart. The arrangement is placed in a uniform *external* electric field **E**, its dipole moment **p** making an angle θ with this field. Two equal and opposite forces **F** and $-\mathbf{F}$ act as shown, where

$$F = qE$$
.

The net force is clearly zero, but there is a net torque about an axis through O (see Eq. 12-2) given by

$$\tau = 2F(a\sin\theta) = 2aF\sin\theta.$$

Combining these two equations and recalling that p = (2a)(q), we obtain

$$\tau = 2aqE\sin\theta = -pE\sin\theta. \tag{27-10}$$

Thus an electric dipole placed in an external electric field \mathbf{E} experiences a torque tensing to align it with the field. Equation 27-10 can be written in vector form as

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E},\tag{27-11}$$

the appropriate vectors being shown in Fig. 27-15b.

Work (positive or negative) must be done by an external agent to change the orientation of an electric dipole in an external field. This work is stored as potential energy U in the system consisting of the dipole and the arrangement used to set up the external field. If θ in Fig. 27-15*a* has the initial value θ_0 , the work required to turn the dipole axis to an angle θ is given (see Table 12-2) from

$$W = \int dW = \int_{\theta_0}^{\theta} \tau \, d\theta = U,$$

where τ is the torque exerted by the agent that does the work. Combining this equation with Eq. 27-10 yields

$$U = \int_{\theta_0}^{\theta} pE\sin\theta \, d\theta = pE \int_{\theta_0}^{\theta} \sin\theta \, d\theta$$
$$= pE \left(-\cos\theta \right) \Big|_{\theta_0}^{\theta}.$$

Since we are interested only in *changes* in potential energy, we can choose the reference orientation θ_0 to have any convenient value, in this case 90°. This gives

$$U = -pE\cos\theta. \tag{27-12}$$

or, in vector terms,

$$U = -\mathbf{p} \cdot \mathbf{E}. \tag{27-13}$$

An electric dipole consists of two opposite charges of magnitude $q = 1.0 \times 10^{-6}$ C separated by d = 2.0 cm. The dipole is placed in an external field of 1.0×10^{5} N/C.

(a) What maximum torque does the field exert on the dipole? The maximum torque is found by putting $\theta = 90^\circ$ in Eq. 27-10 or

$$\tau = pE\sin\theta = qdE\sin\theta$$

= (1.0×10⁻⁶ C)(0.020 m)(1.0×10⁵ N/C)(sin90°)
= 2.0×10⁻³ N·m

(b) How much work must an external agent do to turn the dipole end for end, starting from a position of alignment ($\theta = 0^{\circ}$)? The work is the difference in potential energy U between the positions $\theta = 180^{\circ}$ and $\theta = 0^{\circ}$. From Eq. 27-12, potential energy U

$$W = U_{180^{\circ}} - U_{0^{\circ}} = (-pE\cos 180^{\circ}) - (-pE\cos 0^{\circ})$$

= 2 pE = 2qdE
= (2)(1.0 × 10⁻⁶ C)(0.020 m)(1.0 × 10⁵ N/C)
= 4.0 × 10⁻³ J.

- 1. Name as many scalar fields and vector fields as you can.
- 2. (*a*) In the gravitational attraction between the earth and a stone, can we say that the earth lies in the gravitational field of the stone? (*b*) How is the gravitational field due to the stone related to that due to the earth?
- 3. A positively charged ball hangs from a long silk thread. We wish to measure **E** at a point in the same horizontal plane as that of the hanging charge. To do so, we put a positive test charge q_0 at the point and measure F/q_0 . Will F/q_0 be less than, equal to, or greater than *E* at the point in question?
- 4. Taking into account the quantization of electric charge (the single electron providing the basic charge unit), how can we justify the procedure suggested by Eq. 27-3?
- 5. In exploring electric fields with a test charge we have often assumed, for convenience, that the test charge was positive. Does this really make any difference in determining the field? Illustrate in a simple case of your own devising.
- 6. Electric lines of force never cross. Why?
- 7. In Fig. 27-4 why do the lines of force around the edge of the figure appear, when extended backward, to radiate uniformly from the center of the figure?
- 8. Figure 27-2 shows that **E** has the same value for all points in front of an infinite uniformly charged sheet. Is this reasonable? One might think that the field should be stronger near the sheet because the charges are so much closer?
- 9. If a point charge q of mass m is released from rest in a non-uniform field, will it follow a line of force?

- 10. A point charge is moving in an electric field at right angles to the lines of force. Does any force **F** act on it?
- 11. In Fig. 27-7 path *AB* is not a line of force. How can you tell?
- 12. In Fig. 27-6, why should grass seeds line up with electric lines of force? Grass seeds normally carry no electric charge. See ``Demonstration of the Electric Fields of Current-Carrying Conductors'' by O. Jefimenko, *American Journal of Physics*. January 1962.
- 13. Two point charges of unknown magnitude and sign are a distance *d* apart. The electric field is zero at one point between them, on the line joining them. What can you conclude about the charges?
- 14. Compare the way *E* varies with *r* for (*a*) a point charge (Eq. 27-4), (*b*) a dipole (Eq. 27-8*a*), and (*c*) a quadruple (Problem 33).
- 15. Charges +Q and -Q are fixed a distance L apart and a long straight line is drawn through them. What is the direction of **E** on this line for points (a) between the charges, (b) outside the charges in the direction of +Q, and (c) outside the charges in the direction of -Q?
- 16. Two point charges of unknown sign and magnitude are fixed a distance L apart. Can we have $\mathbf{E} = 0$ for off-axis points (excluding ∞)? Explain.
- 17. In what way does Eq. 27-8*b* fail to represent the lines of force of Fig. 27-5 if we relax the requirement that $r \gg a$?
- 18. If two dipoles of moments \mathbf{p}_1 and \mathbf{p}_2 are superimposed, is the dipole moment of the resulting configuration given by $\mathbf{p}_1 + \mathbf{p}_2$?
- 19. In Fig. 27-5 the force on the lower charge points up and is finite. The crowding of the lines of force, however, suggests that E is infinitely great at the site of this (point) charge. A charge immersed in an infinitely great field should have an infinitely great force acting on it. What is the solution to this dilemma?
- 20. An electric dipole is placed in a *nonuniform* electric field. Is there a net force on it?
- 21. An electric dipole is placed at rest in a uniform external electric field, as in Fig. 27-15a, and released. Discuss its motion.
- 22. An electric dipole has its dipole moment **p** aligned with a uniform external electric field **E**. (*a*) Is the equilibrium stable or unstable? (*b*) Discuss the nature of the equilibrium if **p** and **E** point in opposite directions.