1

ELECTROSTATICS: CHARGES AND FIELDS

ELECTRIC CHARGE

1.1 Electricity appeared to its early investigators as an extraordinary phenomenon. To draw from bodies the "subtle fire," as it was sometimes called, to bring an object into a highly electrified state, to produce a steady flow of current, called for skillful contrivance. Except for the spectacle of lightning, the ordinary manifestations of nature, from the freezing of water to the growth of a tree, seemed to have no relation to the curious behavior of electrified objects. We know now that electrical forces largely determine the physical and chemical properties of matter over the whole range from atom to living cell. For this understanding we have to thank the scientists of the nineteenth century, Ampère, Faraday, Maxwell, and many others, who discovered the nature of electromagnetism, as well as the physicists and chemists of the twentieth century who unraveled the atomic structure of matter.

Classical electromagnetism deals with electric charges and currents and their interactions as if all the quantities involved could be measured independently, with unlimited precision. Here *classical* means simply "nonquantum." The quantum law with its constant h is ignored in the classical theory of electromagnetism, just as it is in ordinary mechanics. Indeed, the classical theory was brought very nearly to its present state of completion before Planck's discovery. It has survived remarkably well. Neither the revolution of quantum physics nor the development of special relativity dimmed the luster of the electromagnetic field equations Maxwell wrote down 100 years ago.

Of course the theory was solidly based on experiment, and because of that was fairly secure within its original range of application—to coils, capacitors, oscillating currents, and eventually radio waves and light waves. But even so great a success does not guarantee validity in another domain, for instance, the inside of a molecule.

Two facts help to explain the continuing importance in modern physics of the classical description of electromagnetism. First, special relativity required no revision of classical electromagnetism. Historically speaking, special relativity *grew out of* classical electromagnetic theory and experiments inspired by it. Maxwell's field equations, developed long before the work of Lorentz and Einstein, proved to be entirely compatible with relativity. Second, quantum modifications of the electromagnetic forces have turned out to be unimportant down to distances less than 10^{-10} centimeters (cm), 100 times smaller than the atom. We can describe the repulsion and attraction of particles in the atom using the same laws that apply to the leaves of an electroscope, although we need quantum mechanics to predict how the particles will behave under those forces. For still smaller distances, a fusion of electromagnetic theory and quantum theory, called *quantum electrodynamics,* has been remarkably successful. Its predictions are confirmed by experiment down to the smallest distances yet explored.

It is assumed that the reader has some acquaintance with the elementary facts of electricity. We are not going to review all the experiments by which the existence of electric charge was demonstrated, nor shall we review all the evidence for the electrical constitution of matter. On the other hand, we do want to look carefully at the experimental foundations of the basic laws on which all else depends. In this chapter we shall study the physics of stationary electric charges—*electrostatics.*

Certainly one fundamental property of electric charge is its existence in the two varieties that were long ago named *positive* and *negative.* The observed fact is that all charged particles can be divided into two classes such that all members of one class repel each other, while attracting members of the other class. If two small electrically charged bodies *A* and *B*, some distance apart, attract one another, and if *A* attracts some third electrified body *C*, then we always find that *B* repels *C*. Contrast this with gravitation: There is only one kind of gravitational mass, and every mass attracts every other mass.

One may regard the two kinds of charge, positive and negative, as opposite manifestations of one quality, much as *right* and *left* are the two kinds of handedness. Indeed, in the physics of elementary particles, questions involving the sign of the charge are sometimes linked to a question of handedness, and to another basic symmetry, the relation of a sequence of events, *a,* then *b,* then *c,* to the temporally reversed sequence *c,* then *b,* then *a.* It is only the duality of electric charge that concerns us here. For every kind of particle in nature, as far as we know, there can exist an *antiparticle,* a sort of electrical "mirror image." The antiparticle carries charge of the opposite sign. If any other intrinsic quality of the particle has an opposite, the antiparticle has that too, whereas in a property which admits no opposite, such as mass, the antiparticle and particle are exactly alike. The electron's charge is negative; its antiparticle, called a *positron,* has a positive charge, but its mass is precisely the same as that of the electron. The proton's antiparticle is called simply an *antiproton;* its electric charge is negative. An electron and a proton combine to make an ordinary hydrogen atom. A positron and an antiproton could combine in the same way to make an atom of antihydrogen. Given the building blocks, positrons, antiprotons, and antineutrons,† there could be built up the whole range of antimatter, from antihydrogen to antigalaxies. There is a practical difficulty, of course. Should a positron meet an electron or an antiproton meet a proton, that pair of particles will quickly vanish in a burst of radiation. It is therefore not surprising that even positrons and antiprotons, not to speak of antiatoms, are exceedingly rare and short-lived in our world. Perhaps the universe contains,

†Although the electric charge of each is zero, the neutron and its antiparticle are not interchangeable. In certain properties that do not concern us here, they are opposite.

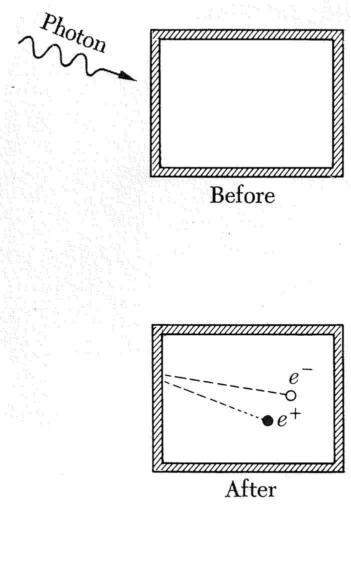
somewhere, a vast concentration of antimatter. If so, its whereabouts is a cosmological mystery.

The universe around us consists overwhelmingly of matter, not antimatter. That is to say, the abundant carriers of negative charge are electrons, and the abundant carriers of positive charge are protons. The proton is nearly 2000 times heavier than the electron and very different, too, in some other respects. Thus matter at the atomic level incorporates negative and positive electricity in quite different ways. The positive charge is all in the atomic nucleus, bound within a massive structure no more than 10^{-12} cm in size, while the negative charge is spread, in effect, through a region about 10^4 times larger in dimensions. It is hard to imagine what atoms and molecules—and all of chemistry—would be like, if not for this fundamental electrical asymmetry of matter.

What we call negative charge, by the way, could just as well have been called positive. The name was a historical accident. There is nothing essentially negative about the charge of an electron. It is not like a negative integer. A negative integer, once multiplication has been defined, differs essentially from a positive integer in that its square is an integer of opposite sign. But the product of two charges is not a charge; there is no comparison.

Two other properties of electric charge are essential in the electrical structure of matter: Charge is *conserved,* and charge is *quantized.* These properties involve *quantity* of charge and thus imply a measurement of charge. Presently we shall state precisely how charge can be measured in terms of the force between charges a certain distance apart, and so on. But let us take this for granted for the time being, so that we may talk freely about these fundamental facts.

FIGURE 1.1
Charged particles are created in pairs with equal and opposite charge.



CONSERVATION OF CHARGE

1.2 The total charge in an isolated system never changes. By *isolated* we mean that no matter is allowed to cross the boundary of the system. We could let light pass into or out of the system, since the "particles" of light, called *photons,* carry no charge at all. Within the system charged particles may vanish or reappear, but they always do so in pairs of equal and opposite charge. For instance, a thin-walled box in a vacuum exposed to gamma rays might become the scene of a "pair-creation" event in which a high-energy photon ends its existence with the creation of an electron and a positron (Fig. 1.1). Two electrically charged particles have been newly created, but the net change in total charge, in and on the box, is zero. An event that *would* violate the law we have just stated would be the creation of a positively charged particle *without* the simultaneous creation of a negatively charged particle. Such an occurrence has never been observed.

Of course, if the electric charges of an electron and a positron were not precisely equal in magnitude, pair creation would still violate the strict law of charge conservation. That equality is a manifestation of the particle–antiparticle duality already mentioned, a universal symmetry of nature.

One thing will become clear in the course of our study of electromagnetism: Nonconservation of charge would be quite incompatible with the structure of our present electromagnetic theory. We may therefore state, either as a postulate of the theory or as an empirical law supported without exception by all observations so far, the charge conservation law:

> The total electric charge in an isolated system, that is, the algebraic sum of the positive and negative charge present at any time, never changes.

Sooner or later we must ask whether this law meets the test of relativistic invariance. We shall postpone until Chapter 5 a thorough discussion of this important question. But the answer is that it does, and not merely in the sense that the statement above holds in any given inertial frame but in the stronger sense that observers in different frames, measuring the charge, obtain the same number. In other words the total electric charge of an isolated system is a relativistically invariant number.

QUANTIZATION OF CHARGE

1.3 The electric charges we find in nature come in units of one magnitude only, equal to the amount of charge carried by a single electron. We denote the magnitude of that charge by e. (When we are paying attention to sign, we write $-e$ for the charge on the electron itself.) We have already noted that the positron carries precisely that amount of charge, as it must if charge is to be conserved when an electron and a positron annihilate, leaving nothing but light. What seems more remarkable is the apparently exact equality of the charges carried by all other charged particles—the equality, for instance, of the positive charge on the proton and the negative charge on the electron.

That particular equality is easy to test experimentally. We can see whether the net electric charge carried by a hydrogen molecule, which consists of two protons and two electrons, is zero. In an experiment carried out by J. G. King,† hydrogen gas was compressed into

†J. G. King, *Phys. Rev. Lett.* **5**:562 (1960). References to previous tests of charge equality will be found in this article and in the chapter by V. W. Hughes in "Gravitation and Relativity," H. Y. Chieu and W. F. Hoffman (eds.), W. A. Benjamin, New York, 1964, chap. 13.

a tank that was electrically insulated from its surroundings. The tank contained about 5×10^{24} molecules [approximately 17 grams (gm)] of hydrogen. The gas was then allowed to escape by means which prevented the escape of any ion—a molecule with an electron missing or an extra electron attached. If the charge on the proton differed from that on the electron by, say, one part in a billion, then each hydrogen molecule would carry a charge of $2 \times 10^{-9}e$, and the departure of the whole mass of hydrogen would alter the charge of the tank by $10^{16}e$, a gigantic effect. In fact, the experiment could have revealed a residual molecular charge as small as $2 \times 10^{-20}e$, and none was observed. This proved that the proton and the electron do not differ in magnitude of charge by more than 1 part in 10^{20}.

Perhaps the equality is really *exact* for some reason we don't yet understand. It may be connected with the possibility, suggested by recent theories, that a proton can, *very* rarely, decay into a positron and some uncharged particles. If that were to occur, even the slightest discrepancy between proton charge and positron charge would violate charge conservation. Several experiments designed to detect the decay of a proton have not yet, as this is written in 1983, registered with certainty a single decay. If and when such an event is observed, it will show that exact equality of the magnitude of the charge of the proton and the charge of the electron (the positron's antiparticle) can be regarded as a corollary of the more general law of charge conservation.

That notwithstanding, there is now overwhelming evidence that the *internal* structure of all the strongly interacting particles called *hadrons*—a class which includes the proton and the neutron—involves basic units called *quarks,* whose electric charges come in multiples of $e/3$. The proton, for example, is made with three quarks, two of charge $\frac{2}{3}e$ and one with charge $-\frac{1}{3}e$. The neutron contains one quark of charge $\frac{2}{3}e$ and two quarks with charge $-\frac{1}{3}e$.

Several experimenters have searched for single quarks, either free or attached to ordinary matter. The fractional charge of such a quark, since it cannot be neutralized by any number of electrons or protons, should betray the quark's presence. So far no fractionally charged particle has been conclusively identified. There are theoretical grounds for suspecting that the liberation of a quark from a hadron is impossible, but the question remains open at this time.

The fact of charge quantization lies outside the scope of classical electromagnetism, of course. We shall usually ignore it and act as if our point charges q could have any strength whatever. This will not get us into trouble. Still, it is worth remembering that classical theory cannot be expected to explain the structure of the elementary particles. (It is not certain that present quantum theory can either!) What holds the electron together is as mysterious as what fixes the precise value of its charge. Something more than electrical forces must be involved, for the electrostatic forces between different parts of the electron would be repulsive.

In our study of electricity and magnetism we shall treat the charged particles simply as carriers of charge, with dimensions so small that their extension and structure is for most purposes quite insignificant. In the case of the proton, for example, we know from high-energy scattering experiments that the electric charge does not extend appreciably beyond a radius of 10^{-13} cm. We recall that Rutherford's analysis of the scattering of alpha particles showed that even heavy nuclei have their electric charge distributed over a region smaller than 10^{-11} cm. For the physicist of the nineteenth century a "point charge" remained an abstract notion. Today we are on familiar terms with the atomic particles. The graininess of electricity is so conspicuous in our modern description of nature that we find a point charge less of an artificial idealization than a smoothly varying distribution of charge density. When we postulate such smooth charge distributions, we may think of them as averages over very large numbers of elementary charges, in the same way that we can define the macroscopic density of a liquid, its lumpiness on a molecular scale notwithstanding.

COULOMB'S LAW

1.4 As you probably already know, the interaction between electric charges at rest is described by Coulomb's law: Two stationary electric charges repel or attract one another with a force proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them.

We can state this compactly in vector form:

$$\mathbf{F}_2 = k\,\frac{q_1 q_2 \hat{\mathbf{r}}_{21}}{r_{21}^2} \qquad (1)$$

Here q_1 and q_2 are numbers (scalars) giving the magnitude and sign of the respective charges, $\hat{\mathbf{r}}_{21}$ is the unit vector in the direction† from charge 1 to charge 2, and $\mathbf{F}_2$ is the force acting on charge 2. Thus Eq. 1 expresses, among other things, the fact that like charges repel and unlike attract. Also, the force obeys Newton's third law; that is, $\mathbf{F}_2 = -\mathbf{F}_1$.

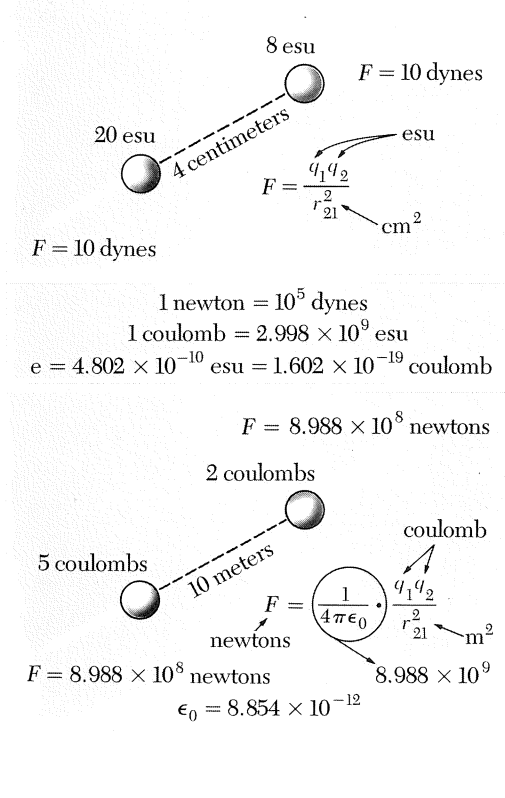
The unit vector $\hat{\mathbf{r}}_{21}$ shows that the force is parallel to the line joining the charges. It could not be otherwise unless space itself has some built-in directional property, for with two point charges alone in empty and isotropic space, no other direction could be singled out.

†The convention we adopt here may not seem the natural choice, but it is more consistent with the usage in some other parts of physics and we shall try to follow it throughout this book.

If the point charge itself had some internal structure, with an axis defining a direction, then it would have to be described by more than the mere scalar quantity q. It is true that some elementary particles, including the electron, do have another property, called *spin.* This gives rise to a magnetic force between two electrons in addition to their electrostatic repulsion. This magnetic force does not, in general, act in the direction of the line joining the two particles. It decreases with the inverse fourth power of the distance, and at atomic distances of 10^{-8} cm the Coulomb force is already about 10^4 times stronger than the magnetic interaction of the spins. Another magnetic force appears if our charges are moving—hence the restriction to stationary charges in our statement of Coulomb's law. We shall return to these magnetic phenomena in later chapters.

Of course we must assume, in writing Eq. 1, that both charges are well localized, each occupying a region small compared with r_{21}. Otherwise we could not even define the distance r_{21} precisely.

FIGURE 1.2
Coulomb's law expressed in CGS electrostatic units (top) and in SI units (bottom). The constant ϵ_0 and the factor relating coulombs to esu are connected, as we shall learn later, with the speed of light. We have rounded off the constants in the figure to four-digit accuracy. The precise values are given in Appendix E.



The value of the constant k in Eq. 1 depends on the units in which r, $\mathbf{F}$, and q are to be expressed. Usually we shall choose to measure r_{21} in cm, $\mathbf{F}$ in dynes, and charge in electrostatic units (esu). Two like charges of 1 esu each repel one another with a force of 1 dyne when they are 1 cm apart. Equation 1, with $k = 1$, is the definition of the unit of charge in CGS electrostatic units, the dyne having already been defined as the force that will impart an acceleration of one centimeter per second per second to a one-gram mass. Figure 1.2*a* is just a graphic reminder of the relation. The magnitude of e, the fundamental quantum of electric charge, is 4.8023×10^{-10} esu.

We want to be familiar also with the unit of charge called the *coulomb.* This is the unit for electric charge in the *Système Internationale* (SI) family of units. That system is based on the meter, kilogram, and second as units of length, mass, and time, and among its electrical units are the familiar volt, ohm, ampere, and watt.

The SI unit of force is the newton, equivalent to exactly 10^5 dynes, the force that will cause a one-kilogram mass to accelerate at one meter per second per second. The coulomb is defined by Eq. 1 with $\mathbf{F}$ in newtons, r_{21} in meters, charges q_1 and q_2 in coulombs, and $k = 8.988 \times 10^9$. A charge of 1 coulomb equals 2.998×10^9 esu. Instead of k, it is customary to introduce a constant ϵ_0, which is just $(4\pi k)^{-1}$, with which the same equation is written

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2 \hat{\mathbf{r}}_{21}}{r_{21}^2} \qquad (1')$$

Refer to Fig. 1.2*b* for an example. The constant ϵ_0 will appear in several SI formulas that we'll meet in the course of our study. The exact value of ϵ_0 and the exact relation of the coulomb to the esu can be found in Appendix E. For our purposes the following approximations are quite accurate enough: $k = 9 \times 10^9$; 1 coulomb $= 3 \times 10^9$ esu.

Fortunately the electronic charge e is very close to an easily remembered approximate value in either system: $e = 4.8 \times 10^{-10}$ esu $= 1.6 \times 10^{-19}$ coulomb.

The only way we have of detecting and measuring electric charges is by observing the interaction of charged bodies. One might wonder, then, how much of the apparent content of Coulomb's law is really only definition. As it stands, the significant physical content is the statement of inverse-square dependence and the implication that electric charge is *additive* in its effect. To bring out the latter point, we have to consider *more* than two charges. After all, if we had only two charges in the world to experiment with, q_1 and q_2, we could never measure them separately. We could verify only that F is proportional to $1/r_{21}^2$. Suppose we have *three* bodies carrying charges q_1, q_2, and q_3. We can measure the force on q_1 when q_2 is 10 cm away from q_1 and q_3 is very far away, as in Fig. 1.3*a*. Then we can take q_2 away, bring q_3 into q_2's former position, and again measure the force on q_1. Finally, we bring q_2 and q_3 very close together and locate the combination 10 cm from q_1. We find by measurement that the force on q_1 is equal to the sum of the forces previously measured. This is a significant result that could *not* have been predicted by logical arguments from symmetry like the one we used above to show that the force between two point charges *had* to be along the line joining them. *The force with which two charges interact is not changed by the presence of a third charge.*

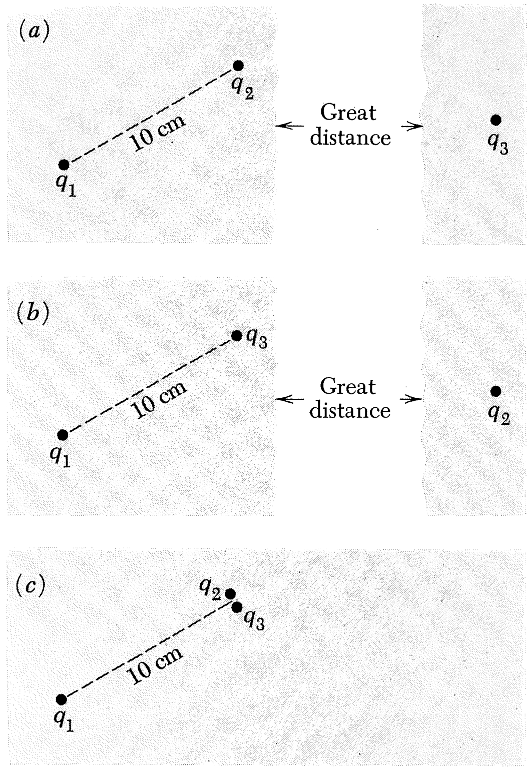


FIGURE 1.3
The force on q_1 in (*c*) is the sum of the forces on q_1 in (*a*) and (*b*).

No matter how many charges we have in our system. Coulomb's law (Eq. 1) can be used to calculate the interaction of every pair. This is the basis of the principle of *superposition,* which we shall invoke again and again in our study of electromagnetism. Superposition means combining two sets of sources into one system by adding the second system "on top of" the first without altering the configuration of either one. Our principle ensures that the force on a charge placed at any point in the combined system will be the vector sum of the forces that each set of sources, acting alone, causes to act on a charge at that point. This principle must not be taken lightly for granted. There may well be a domain of phenomena, involving very small distances or very intense forces, where superposition *no longer holds.* Indeed, we know of quantum phenomena in the electromagnetic field which do represent a failure of superposition, seen from the viewpoint of the classical theory.

Thus the physics of electrical interactions comes into full view only when we have *more* than two charges. We can go beyond the explicit statement of Eq. 1 and assert that, with the three charges in Fig. 1.3 occupying any positions whatever, the force on any one of them, such as q_3, is correctly given by this equation:

$$\mathbf{F}_3 = \frac{q_3 q_1 \hat{\mathbf{r}}_{31}}{r_{31}^2} + \frac{q_3 q_2 \hat{\mathbf{r}}_{32}}{r_{32}^2} \qquad (2)$$

The experimental verification of the inverse-square law of electrical attraction and repulsion has a curious history. Coulomb himself announced the law in 1786 after measuring with a torsion balance the force between small charged spheres. But 20 years earlier Joseph Priestly, carrying out an experiment suggested to him by Benjamin Franklin, had noticed the absence of electrical influence within a hollow charged container and made an inspired conjecture: "May we not infer from this experiment that the attraction of electricity is subject to the same laws with that of gravitation and is therefore according to the square of the distances; since it is easily demonstrated that were the earth in the form of a shell, a body in the inside of it would not be attracted to one side more than the other."† The same idea was the basis of an elegant experiment in 1772 by Henry Cavendish. Cavendish charged a spherical conducting shell which contained within it, and temporarily connected to it, a smaller sphere. The outer shell was then separated into two halves and carefully removed, the inner sphere having been first disconnected. This sphere was tested for charge, the absence of which would confirm the inverse-square law. Assuming that a deviation from the inverse-square law could be expressed as a difference in the exponent, $2 + \delta$, say, instead of 2, Cavendish concluded that δ must be less than 0.03. This experiment of Cavendish remained largely unknown until Maxwell discovered and published Cavendish's notes a century later (1876). At that time also Maxwell repeated the experiment with improved apparatus, pushing the limit down to $\delta < 10^{-6}$. The latest of several modern versions of the Cavendish experiment,‡ if interpreted the same way, yielded the fantastically small limit $\delta < 10^{-15}$.

During the second century after Cavendish, however, the question of interest changed somewhat. Never mind how perfectly Coulomb's law works for charged objects in the laboratory—is there a range of distances where it completely breaks down? There are two domains in either of which a breakdown is conceivable. The first is the domain of very small distances, distances less than 10^{-14} cm where electromagnetic theory as we know it may not work at all. As for very large distances, from the geographical, say, to the astronomical, a test of Coulomb's law by the method of Cavendish is obviously not feasible. Nevertheless we do observe certain large-scale electromagnetic phenomena which prove that the laws of classical electromagnetism work over very long distances. One of the most stringent tests is provided by planetary magnetic fields, in particular, the magnetic field of the giant planet Jupiter, which was surveyed in the mission of Pioneer

†Joseph Priestly, "The History and Present State of Electricity," vol. II, London, 1767.

‡E. R. Williams, J. G. Faller, and H. Hill. *Phys. Rev. Lett.* **26**:721 (1971).

10. The spatial variation of this field was carefully analyzed† and found to be entirely consistent with classical theory out to a distance of at least 10^5 kilometers (km) from the planet. This is tantamount to a test, albeit indirect, of Coulomb's law over that distance.

To summarize, we have every reason for confidence in Coulomb's law over the stupendous range of 24 decades in distance, from 10^{-14} to 10^{10} cm, if not farther, and we take it as the foundation of our description of electromagnetism.

ENERGY OF A SYSTEM OF CHARGES

1.5 In principle, Coulomb's law is all there is to electrostatics. Given the charges and their locations we can find all the electrical forces. Or given that the charges are free to move under the influence of other kinds of forces as well, we can find the equilibrium arrangement in which the charge distribution will remain stationary. In the same sense, Newton's laws of motion are all there is to mechanics. But in both mechanics and electromagnetism we gain power and insight by introducing other concepts, most notably that of energy.

Energy is a useful concept here because electrical forces are *conservative.* When you push charges around in electric fields, no energy is irrecoverably lost. Everything is perfectly reversible. Consider first the work which must be done *on* the system to bring some charged bodies into a particular arrangement. Let us start with two charged bodies or particles very far apart from one another, as indicated at the top of Fig. 1.4, carrying charges q_1 and q_2. Whatever energy may have been needed to create these two concentrations of charge originally we shall leave entirely out of account. Bring the particles slowly together until the distance between them is r_{12}. How much work does this take?

It makes no difference whether we bring q_1 toward q_2 or the other way around. In either case the work done is the integral of the product: force times displacement in direction of force. The force that has to be applied to move one charge toward the other is equal to and opposite the Coulomb force.

$$W = \int \text{force} \times \text{distance} = \int_{r=\infty}^{r_{12}} \frac{q_1 q_2(-dr)}{r^2} = \frac{q_1 q_2}{r_{12}} \qquad (3)$$

Because r is changing from ∞ to r_{12}, the increment of displacement is $-dr$. We know the work done on the system must be positive for charges of like sign; they have to be pushed together. With q_1 and q_2 in esu, and r_{12} in cm, Eq. 3 gives the work in ergs.

†L. Davis, Jr., A. S. Goldhaber, M. M. Nieto, *Phys. Rev. Lett.* **35**:1402 (1975). For a review of the history of the exploration of the outer limit of classical electromagnetism, see A. S. Goldhaber and M. M. Nieto, *Rev. Mod. Phys.* **43**:277 (1971).

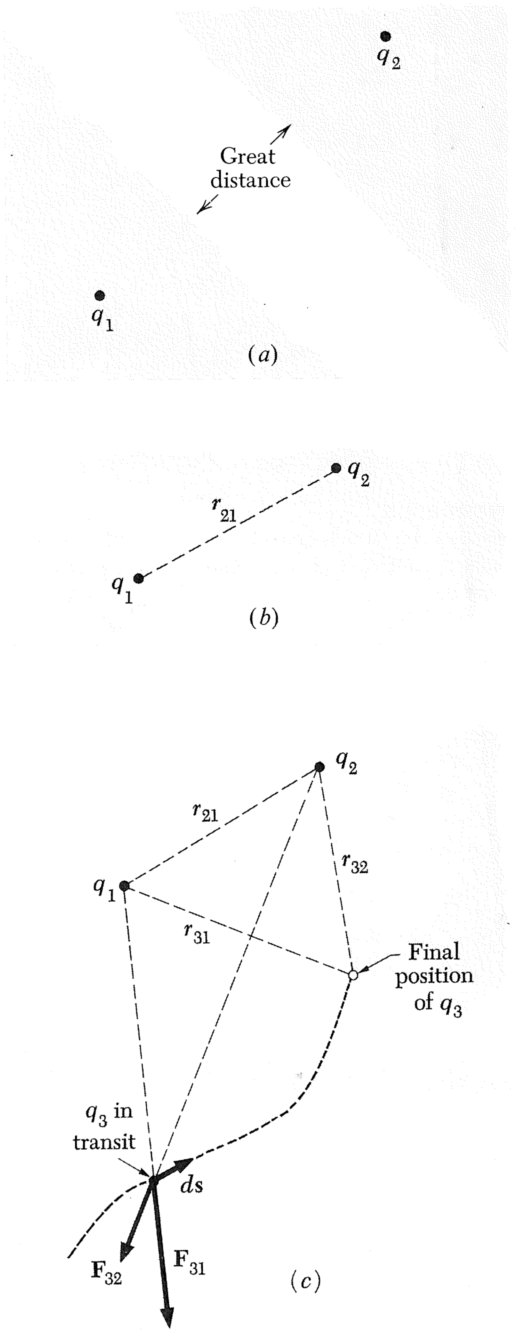


FIGURE 1.4
Three charges are brought near one another. First q_2 is brought in; then with q_1 and q_2 fixed, q_3 is brought in.